Mathematical and Computational Applications, Vol. 11, No. 2, pp. 137-149, 2006.

A COMPARISON BETWEEN ACTIVE AND PASSIVE VIBRATION CONTROL OF NON-LINEAR SIMPLE PENDULUM PART I: TRANSVERSALLY TUNED ABSORBER AND NEGATIVE $G\dot{\varphi}^n$ FEEDBACK

M. Eissa* and M. Sayed
Department of Engineering Mathematics, Faculty of Electronic Engineering Menouf 32952, Egypt., * moh_6_11@yahoo.com

Abstract- Vibrations and dynamic chaos should be controlled in structures and machines. The most famous method of vibration control is using absorbers or tuned dampers or neutralizers. The main aim is to improve the system behavior at different resonance conditions. Two main strategies are used. They are passive and active control methods. In this paper, a tuned absorber, in the transversally direction, is added to an externally excited pendulum, which is simulated by a second order non-linear differential equation having both quadratic and cubic non-linearties, subjected to harmonic excitation. The absorber is usually designed to control one frequency at primary resonance where system damage is probable. The quenching efficiency of the system is studied. Active control is applied to the system via negative velocity feedback or its square or cubic value. The multiple time scale perturbation technique is applied throughout. An approximate solution is derived up to the second order approximation. The stability of the system is investigated applying both frequency response equations and phase plane methods. The effects of the absorber on system behavior are studied numerically. Optimum working conditions of the system are extracted when applying both passive and active control methods.
Keywords- Spring-pendulum, Absorber, Active and passive control.

1. INTRODUCTION

Vibrations and dynamic chaos are undesired phenomenon in structures. They cause disturbance, discomfort, damage and destruction of the system or the structure. For these reasons, money, time and effort are spent to get rid of both vibrations and noise or chaos or to minimize them. One of the most effective tools for passive vibration control is the dynamic absorber or the damper or the neutralizer [1]. Eissa [2] has shown that a non-linear absorber can be used to control the vibration of a non-linear system. Also, he has shown that the non-linear absorber widens its range of applications, and its damping coefficient should be kept minimum for better performance [3]. Cheng-Tang Lee et al. [4] demonstrated a dynamic vibration absorber system, which can be used to reduce speed fluctuations in rotating machinery. Eissa and El-Ganaini [5,6] studied the control of both vibration and dynamic chaos of both internal combustion engines and mechanical structures having quadratic and cubic non-linearties, subjected to harmonic excitation using single and multi-absorbers. Active constrained layer damping (ACLD) has been successfully utilized as effective means of damping out the vibration of various flexible structures [7-12]. A variable stiffness vibration absorber without damping is used for controlling the principal mode of a vibrating structure. The optimal vibration absorber is also utilized for controlling higher mode [13]. Another approach of active damping of mechanical structures is the hybrid system, which is a combination of semi-active and active treatments, in which the advantages of individual schemes are combined, while eliminating their shortcomings [14]. Active damping of mechanical structures can be utilized using piezoceramic sensors and actuators [15-16]. The vibration of rotating machinery is suppressed by eliminating the root cause of the vibration system imbalance [17].

In the present paper, a tuned absorber, which can move in the transversally direction, is added to an externally excited pendulum, which is described by a second order non-linear differential equation having both quadratic and cubic non-linearties, subjected to harmonic excitation. Active control is applied to the system via negative velocity feedback or its square or cubic value. The multiple time scale perturbation technique is applied throughout. An approximate solution is derived up to second order approximation. The stability of the system is investigated applying both frequency response equations and phase plane methods. The effects of the absorber on system behavior are studied numerically. Optimum working conditions of the system are obtained applying both passive and active control methods. Both control methods are demonstrated numerically.

2. MATHEMATICAL MODELING

The mechanical system is a non-linear simple pendulum consisting of a mass M and a string of length l, and an absorber of mass m elastically suspended to the pendulum. The absorber mass can move transversally in the direction perpendicular to the pendulum axis as shown in Fig. 1. Here φ is the angular displacement of the pendulum and u is the displacement of the absorber mass from its equilibrium position. From the principles of the mechanics, the kinetic and potential energies are given in the following forms respectively:

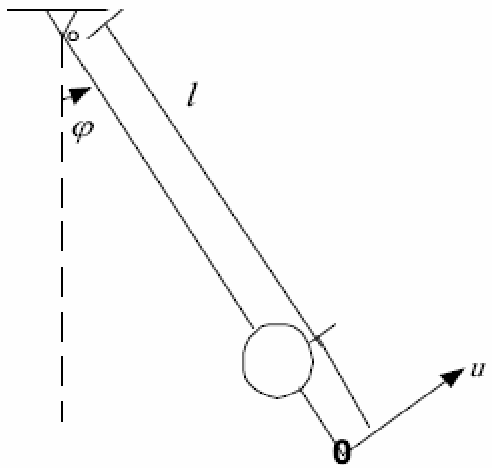


Fig.1. Schematic representation of system

$$T = \frac{1}{2} M l^2 \dot{\varphi}^2 + \frac{1}{2} m\{[(l + l_0)\dot{\varphi} + \dot{u}]^2 + u^2 \dot{\varphi}^2\} \tag{1}$$

$$U = Mg\, l(1 - \cos\varphi) + mg[(l + l_0)(1 - \cos\varphi) + u \sin\varphi] + \frac{1}{2} k_1 u^2 + \frac{1}{4} k_3 u^4 \tag{2}$$

Applying Lagrangian equations and taking into account the effects of linear viscous damping and external excitation of the main system, the following differential equations of motion are obtained:

$$\ddot{\varphi} + c_1\dot{\varphi} + \omega_1^2 \sin\varphi + \beta\{u^2\ddot{\varphi} + (l+l_0)\ddot{u} + 2u\dot{u}\dot{\varphi} + gu\cos\varphi\} = f\cos\Omega t \quad (3)$$

$$\ddot{u} + c_2\dot{u} + \omega_2^2 u + \alpha u^3 + (l+l_0)\ddot{\varphi} - u\dot{\varphi}^2 + g\sin\varphi = 0 \quad (4)$$

where α is the spring stiffness non-linear parameter, ω_1 & ω_2 are the natural frequencies, c_1 & c_2 are the linear damping coefficients of the pendulum and absorber respectively, f is the forcing amplitude and Ω is the forcing frequency of the pendulum. It is assumed that both u and φ are small, and the whole motion is a planer one. Due to these assumptions, both ($\sin\varphi$) and ($\cos\varphi$) can be written in the form:

$$\cos\varphi \cong 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} \qquad \sin\varphi \cong \varphi - \frac{\varphi^3}{3!} \quad (5, 6)$$

The damping coefficients and the forcing amplitudes are assumed to be in the form:

$$c_n = \varepsilon\,\hat{c}_n \;, \; f = \varepsilon^2 \hat{f} \text{ and } \beta = \varepsilon\,\hat{\beta} \qquad n = 1,2. \quad (7)$$

where ε is a small perturbation parameter and $0 < \varepsilon << 1$.
Eqns. (3) and (4) can be re-written in the form:

$$\ddot{\varphi} + \varepsilon\hat{c}_1\dot{\varphi} + \omega_1^2(\varphi - \frac{\varphi^3}{3!}) + \varepsilon\hat{\beta}\{u^2\ddot{\varphi} + (l+l_0)\ddot{u}$$
$$+2u\dot{u}\dot{\varphi} + gu(1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!})\} = \varepsilon^2\hat{f}\cos\omega t \quad (8)$$

$$\ddot{u} + \varepsilon\hat{c}_2\dot{u} + \omega_2^2 u + \alpha u^3 + (l+l_0)\ddot{\varphi} - u\dot{\varphi}^2 + g(\varphi - \frac{\varphi^3}{3!}) = 0 \quad (9)$$

Assuming the solution of equations (8) and (9) to be in the form

$$\varphi(t;\varepsilon) = \varepsilon\varphi_1(T_0,T_1) + \varepsilon^2\varphi_2(T_0,T_1) + ... \quad (10)$$

$$u(t;\varepsilon) = \varepsilon u_1(T_0,T_1) + \varepsilon^2 u_2(T_0,T_1) + ... \quad (11)$$

where $T_n = \varepsilon^n t$, $(n = 0,1)$. The derivatives will be in the forms

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + ... \qquad \frac{d^2}{dt^2} = D_0^{\,2} + 2\varepsilon D_0 D_1 + ... \quad 12, 13)$$

where $Dn = \frac{\partial}{\partial Tn}$, n = 0,1. Equating the similar powers of ε in both side's yields.

$$(D_0^2 + \omega_1^2)\varphi_1 = 0 \quad (14)$$

$$(D_0^2 + \omega_2^2)u_1 = \omega_1^2(l+l_0)\varphi_1 \quad (15)$$

$$(D_0^2 + \omega_1^2)\varphi_2 = -2D_0D_1\varphi_1 + \hat{\beta}(l+l_0)^2 D_0^2\varphi_1 - \hat{c}_1 D_0\varphi_1 + \omega_2^2\hat{\beta}(l+l_0)u_1$$
$$+(\hat{f}/2)\exp(i\Omega T_0) \quad (16)$$

$$(D_0^2 + \omega_2^2)u_2 = -2D_0D_1u_1 + \hat{\beta}(l+l_0)^2 D_0^2 u_1 + \hat{c}_1(l+l_0)D_0\varphi_1 - \omega_1^2(l+l_0)\varphi_2 - \hat{c}_2 D_0 u_1$$
$$-g\varphi_1 - [(l+l_0)\hat{f}/2]\cos\Omega t + \beta g(l+l_0)u_1 \quad (17)$$

The general solutions of Eqns. (14) and (15) can be written in the form

$$\varphi_1 = A_1 \exp(i\,\omega_1 T_0) + cc \tag{18}$$

$$u_1 = A_2 \exp(i\,\omega_2 T_0) + \frac{\omega_1^2 (l + l_0)}{(\omega_2^2 - \omega_1^2)} A_1 \exp(i\,\omega_1 T_0) + cc \tag{19}$$

where A_1 and A_2 are complex functions in T_1, which can be determined from eliminating the secular terms at the next approximation, and *cc* represents the complex conjugates. Substituting from Eqns. (18) and (19) into Eqns. (16) and (17) and eliminating the secular terms, then the first-order approximation is obtained as:

$$\varphi_2 = E_1 \exp(i\omega_1 T_0) + E_2 \exp(i\omega_2 T_0) + E_3 \exp(i\Omega T_0) + cc \tag{20}$$

$$u_2 = C_1 \exp(i\omega_2 T_0) + C_2 \exp(i\omega_1 T_0) + C_3 \exp(i\Omega T_0) + cc \tag{21}$$

where E_j and $C_j\,(j = 1, 2, 3)$ are complex functions in T_1.

From the above-derived solutions, the reported resonance cases are:
a- Primary resonance (1) $\Omega \cong \omega_1$ (2) $\Omega \cong \omega_2$
b- Internal or secondary resonance (1) $\omega_2 \cong \omega_1$
c- Simultaneous or incident resonance
Any combination of the above resonance cases is considered as simultaneous or incident resonance.

3. STABILITY OF THE SYSTEM

Using the simultaneous primary resonance conditions $\Omega = \omega_1 + \varepsilon\hat{\sigma}_1$, $\Omega = \omega_2 + \varepsilon\hat{\sigma}_2$ (where $\sigma_1 = \varepsilon\hat{\sigma}_1$ and $\sigma_2 = \varepsilon\hat{\sigma}_2$ are called detuning parameters) and eliminating the secular terms leads to solvability conditions.

$$2i\,\omega_1 D_1 A_1 = -i\hat{c}_1\omega_1 A_1 + \beta(l + l_0)\omega_1^2 [\frac{\omega_2^2}{(\omega_2^2 - \omega_1^2)} - (l + l_0)] A_1 + \frac{\hat{f}}{2} \exp(i\,\hat{\sigma}_1 T_1) \tag{22}$$

$$2i\,\omega_2 D_1 A_2 = -i\hat{c}_2\omega_2 A_2 + \beta(l + l_0)\omega_2^2 [\frac{\omega_1^2 - g}{(\omega_1^2 - \omega_2^2)} + (l + l_0)] A_2 - \frac{(l + l_0)\hat{f}}{2} \exp(i\,\hat{\sigma}_2 T_1) \tag{23}$$

Putting $A_n = \dfrac{\hat{a}_n}{2} \exp(i\gamma_n)$, $a_n = \varepsilon\hat{a}_n$ n = 1, 2 (24)

where $a_n \& \gamma_n$ are the steady state amplitudes and the phases of the motions respectively. Substituting from equation (24) into equations (22)-(23) and equating the real and imaginary parts we obtained

$$\dot{a}_1 = -\frac{c_1 a_1}{2} + \frac{f}{2\omega_1} \sin(\theta_1) \tag{25}$$

$$a_1\dot{\theta}_1 = a_1\sigma_1 + \beta(l + l_0)\omega_1 [\frac{\omega_2^2}{(\omega_2^2 - \omega_1^2)} - (l + l_0)] \frac{a_1}{2} + \frac{f}{2\omega_1} \cos(\theta_1) \tag{26}$$

$$\dot{a}_2 = -\frac{c_2 a_2}{2} - \frac{(l + l_0) f}{2\omega_2} \sin(\theta_2) \tag{27}$$

$$a_2\dot{\theta}_2 = a_2\sigma_2 + \beta(l + l_0)\omega_2 [\frac{g - \omega_1^2}{(\omega_2^2 - \omega_1^2)} + (l + l_0)] \frac{a_2}{2} - \frac{(l + l_0) f}{2\omega_2} \cos(\theta_2) \tag{28}$$

where $\theta_1 = \hat{\sigma}_1 T_1 - \gamma_1$ *and* $\theta_2 = \hat{\sigma}_2 T_1 - \gamma_2$

The periodic motions are obtained by setting $\dot{a}_n = \dot{\theta}_n = 0$. Hence, the fixed points of Eqns. (25)- (28) are given by

$$-\frac{c_1 a_1}{2} + \frac{f}{2\omega_1}\sin(\theta_1) = 0 \tag{29}$$

$$a_1\sigma_1 + \beta(l+l_0)\omega_1[\frac{\omega_2^2}{(\omega_2^2-\omega_1^2)} - (l+l_0)]\frac{a_1}{2} + \frac{f}{2\omega_1}\cos(\theta_1) = 0 \tag{30}$$

$$-\frac{c_2 a_2}{2} - \frac{(l+l_0)f}{2\omega_2}\sin(\theta_2) = 0 \tag{31}$$

$$a_2\sigma_2 + \beta(l+l_0)\omega_2[\frac{g-\omega_1^2}{(\omega_2^2-\omega_1^2)} + (l+l_0)]\frac{a_2}{2} - \frac{(l+l_0)f}{2\omega_2}\cos(\theta_2) = 0 \tag{32}$$

There are three possibilities in addition to the trivial solution. They are:

(1) $a_1 \neq 0$, $a_2 = 0$ **(2)** $a_2 \neq 0$, $a_1 = 0$ **(3)** $a_1 \neq 0$, $a_2 \neq 0$

Case (1): In this case, we take $a_2 = 0$ and the frequency response equation is given by

$$a_1^2\sigma_1^2 + \frac{c_1^2 a_1^2}{4} + \beta^2(l+l_0)^2\omega_1^2[\frac{\omega_2^2}{(\omega_2^2-\omega_1^2)} - (l+l_0)]^2\frac{a_1^2}{4}$$
$$+\sigma_1\beta(l+l_0)\omega_1[\frac{\omega_2^2}{(\omega_2^2-\omega_1^2)} - (l+l_0)]a_1^2 - \frac{f^2}{4\omega_1^2} = 0 \tag{33}$$

Case (2): In this case, we take $a_1 = 0$ and the frequency response equation is given by

$$a_2^2\sigma_2^2 + \frac{c_2^2 a_2^2}{4} + \beta^2(l+l_0)^2\omega_2^2[\frac{g-\omega_1^2}{(\omega_2^2-\omega_1^2)} + (l+l_0)]^2\frac{a_2^2}{4}$$
$$+\sigma_2\beta(l+l_0)\omega_2[\frac{g-\omega_1^2}{(\omega_2^2-\omega_1^2)} + (l+l_0)]a_2^2 - \frac{(l+l_0)^2 f^2}{4\omega_2^2} = 0 \tag{34}$$

Case (3): The frequency response equations are given by

$$a_1^2\sigma_1^2 + \frac{c_1^2 a_1^2}{4} + \beta^2(l+l_0)^2\omega_1^2[\frac{\omega_2^2}{(\omega_2^2-\omega_1^2)} - (l+l_0)]^2\frac{a_1^2}{4}$$
$$+\sigma_1\beta(l+l_0)\omega_1[\frac{\omega_2^2}{(\omega_2^2-\omega_1^2)} - (l+l_0)]a_1^2 - \frac{f^2}{4\omega_1^2} = 0 \tag{35}$$

$$a_2^2\sigma_2^2 + \frac{c_2^2 a_2^2}{4} + \beta^2(l+l_0)^2\omega_2^2[\frac{g-\omega_1^2}{(\omega_2^2-\omega_1^2)} + (l+l_0)]^2\frac{a_2^2}{4}$$
$$+\sigma_2\beta(l+l_0)\omega_2[\frac{g-\omega_1^2}{(\omega_2^2-\omega_1^2)} + (l+l_0)]a_2^2 - \frac{(l+l_0)^2 f^2}{4\omega_2^2} = 0 \tag{36}$$

3.1 Stability of the fixed points

To analyze the stability of the fixed points, one lets

$$a_n = a_{n0} + a_{n1}, \theta_n = \theta_{n0} + \theta_{n1} \tag{37}$$

where a_{n0}, θ_{n0} are the solutions of Eqns. (29)-(32). Inserting Eq. (37) into Eqns. (25)-(28) and keeping only the linear terms in a_{n1}, θ_{n1}, we get

$$\dot{a}_{11} = -\frac{c_1 a_{11}}{2} + [\frac{f}{2\omega_1}\cos(\theta_{10})]\theta_{11} \tag{38}$$

$$\dot{\theta}_{11} = \{\frac{\sigma_1}{a_{10}} + \frac{\beta(l+l_0)\omega_1}{2a_{10}}[\frac{\omega_2^2}{(\omega_2^2-\omega_1^2)} - (l+l_0)]\}a_{11} - \frac{f}{2\omega_1 a_{10}}\sin(\theta_{10})\,\theta_{11} \tag{39}$$

$$\dot{a}_{21} = -\frac{c_2 a_{21}}{2} - \frac{(l+l_0)f}{2\omega_2}\cos(\theta_{20})\,\theta_{21} \tag{40}$$

$$\dot{\theta}_{21} = \{\frac{\sigma_2}{a_{20}} + \frac{\beta(l+l_0)\omega_2}{2a_{20}}[\frac{g-\omega_1^2}{(\omega_2^2-\omega_1^2)} + (l+l_0)]\}a_{21} + \frac{(l+l_0)f}{2\omega_2 a_{20}}\sin(\theta_{20})\,\theta_{21} \tag{41}$$

The stability of a particular fixed point with respect to perturbations proportional to exp (λT_1) depends on the real parts of the roots of the matrix. Thus a fixed point given in Eqns. (38)-(41) is an asymptotically stable if and only if the real parts of all roots of the matrix are negative.

To study the stability of the fixed points corresponding to case (1), we let $a_{21} = \theta_{21} = 0$ in Eqns. (38)-(41), and obtain the eigenvalues

$$\lambda = (L_1 + L_4) \pm \sqrt{(L_1 + L_4)^2 + 4(L_2 L_3 - L_1 L_4)} \tag{42}$$

where $L_1 = -\frac{c_1}{2}$, $L_2 = \frac{f}{2\omega_1}\cos\theta_1$

$$L_3 = \frac{\sigma_1}{a_1} + \frac{\beta(l+l_0)\omega_1}{2a_1}[\frac{\omega_2^2}{(\omega_2^2-\omega_1^2)} - (l+l_0)] \text{ and } L_4 = -\frac{f}{2\omega_1 a_1}\sin(\theta_1) \tag{43}$$

And hence the fixed points are unstable if and only if

$$L_2 L_3 > L_1 L_4 \tag{44}$$

Otherwise they are stable.

To study the stability of the fixed points corresponding to case (2), we let $a_{11} = \theta_{11} = 0$ in Eqns. (38)-(41), and obtain the eigenvalues

$$\lambda = (L_5 + L_8) \pm \sqrt{(L_5 + L_8)^2 + 4(L_6 L_7 - L_5 L_8)} \tag{45}$$

where $L_5 = -\frac{c_2}{2}$, $L_6 = \frac{-(l+l_0)f}{2\omega_2}\cos\theta_2$

$$L_7 = \frac{\sigma_2}{a_2} + \frac{\beta(l+l_0)\omega_2}{2a_2}[\frac{g-\omega_1^2(l+l_0)}{(\omega_2^2-\omega_1^2)} + (l+l_0)] \quad \text{and } L_8 = -\frac{(l+l_0)f}{2\omega_2 a_2}\sin(\theta_2) \tag{46}$$

And hence the fixed points are unstable if and only if

$$L_6 L_7 > L_5 L_8 \tag{47}$$

Otherwise they are stable.

For the stability of the fixed points corresponding to case (3), the eigenvalues are given by the equation

$$\lambda^4 + R_1\lambda^3 + R_2\lambda^2 + R_3\lambda + R_4 = 0 \tag{48}$$

where R_1, R_2, R_3 and R_4 are functions of the parameters (a_1, a_2, ω_1, ω_2, σ_1, σ_2, f, θ_1, θ_2). According to the Routh-Huriwitz criterion, the necessary and sufficient conditions for all the roots of Eq. (48) to possess negative real parts is that

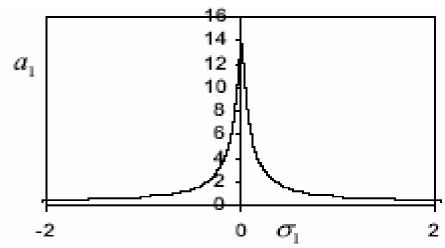
$$R_1 > 0,\ R_1R_2 - R_3 > 0,\ R_3(R_1R_2 - R_3) - R_1^2R_4 > 0,\ R_4 > 0 \tag{49}$$

4. RESULTS AND DISCUSSIONS

Results are presented in graphical forms as steady state amplitudes against detuning parameters and as time history or the response for both pendulum and absorber. A good criterion of both stability and dynamic chaos is the phase-plane trajectories, which are shown for some cases. In the following sections, the effects of the different parameters on response and stability will be investigated. Also different primary resonance cases are studied and discussed.

4.1 System stability

Fig. 2a, shows the effects of the detuning parameter σ_1 on the steady state amplitude of the main system a_1 for the stability first case, where $a_1 \neq 0$ and $a_2 = 0$. It can be seen from the figure that the maximum steady state amplitude occurs at primary resonance when $\Omega \cong \omega_1$. Fig. 2b shows that the steady state amplitude of the main system is a monotonic increasing function in the excitation amplitude f.



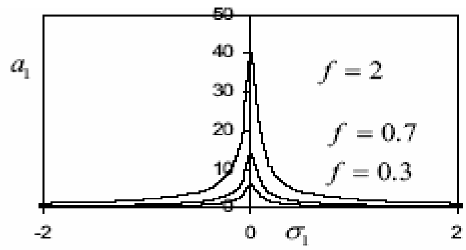


Fig.2a. Effects of the detuning parameter σ_1 *Fig.2b. Effects of excitation amplitude f*

$\omega_1 = 0.5; \omega_2 = 0.8; f = 0.7;$
$c_1 = 0.1;\ \beta = 0.003.$

Fig. 2c shows that the steady state amplitude of the main system is a monotonic decreasing function in its natural frequency ω_1. Fig. 2d shows that for positive and negative values of the non-linear parameter β there exits a shift of the curve to the right and left respectively. The shift means that the equivalent natural frequency is either increased ($\beta > 0$) or decreased ($\beta < 0$). Fig. 2e shows that the steady state amplitude of the main system is a monotonic decreasing function in the damping coefficient c_1.

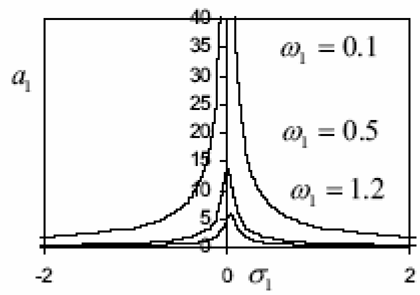


Fig.2c. Effects of the natural frequency ω_1

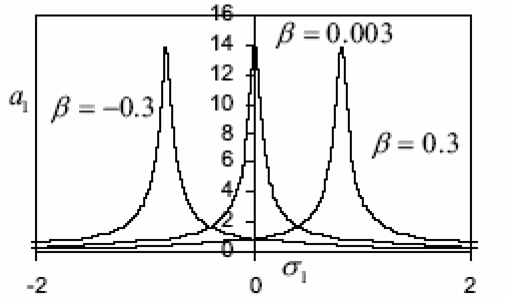


Fig.2d. Effects of the non-linear parameter β

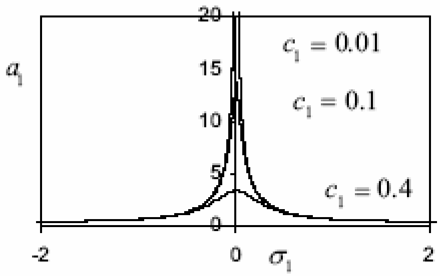


Fig.2e. Effects of damping coefficient c_1

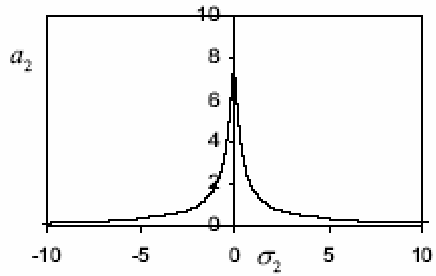


Fig.3a. Effects of the detuning parameter σ_2

$\omega_1 = 0.5$; $\omega_2 = 0.8$; $f = 0.7$;
$c_2 = 0.1$; $\beta = 0.003$.

Fig. 3a, shows the effects of the detuning parameter σ_2 on the steady state amplitude of the absorber a_2 for the stability second case, where $a_1 = 0$ and $a_2 \neq 0$. It can be seen from the figure that the maximum steady state amplitude occurs at primary resonance where $\Omega \cong \omega_2$. Figs. (3b-3c) shows that the steady state amplitude of the absorber is a monotonic increasing function to the excitation amplitude f and natural frequency ω_1. It is worth to notice that as ω_1 is a way from ω_2, and then the effectiveness of the absorber is decreased due to less energy transfer from the main system to the absorber. Figs. (3d-3f) shows that the steady state amplitude of the absorber is a monotonic decreasing function in the natural frequency ω_2, damping coefficient c_2 and the non-linear parameter β.

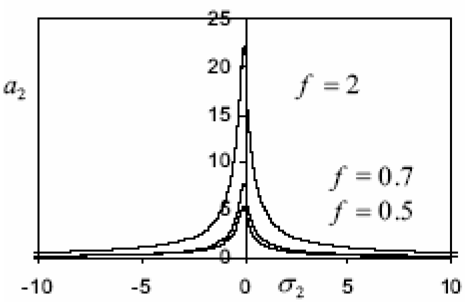


Fig.3b. Effects of excitation amplitude f

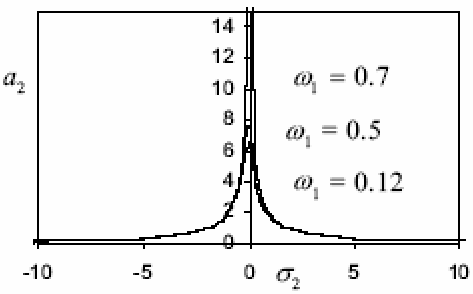
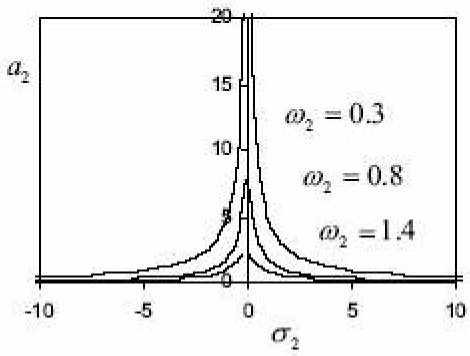


Fig.3c. Effects of the natural frequency ω_1



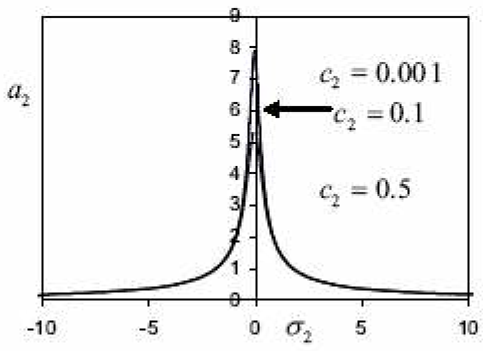
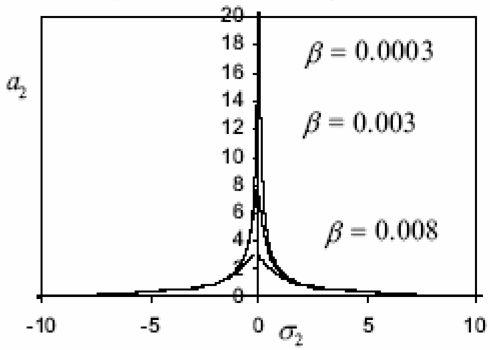


Fig.3d. Effects of the natural frequency ω_2 *Fig.3e. Effects of damping coefficient c_2*

Fig. 4 shows the effect of the non-linear parameter α on the main system. From the figure, we can see that the steady state amplitude is a monotonic increasing function in α. The equivalent natural frequency of the absorber increases as α is increased. This means that less energy transfer from the system to the absorber. The effects on the absorber response are non-significant.



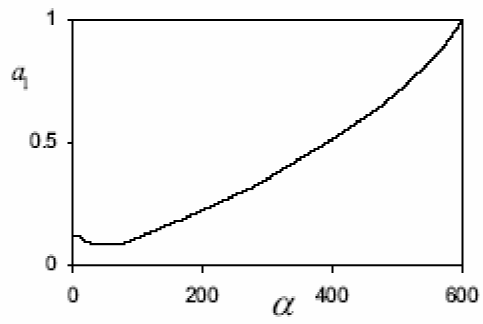


Fig.3f. Effects of the non-linear parameter β *Fig.4. Effects of the non-linear parameter α*

4.2 Passive control

In the following section the effects of the absorber on pendulum response, stability and dynamic chaos are discussed. As shown in Fig. 5 the steady state amplitude without absorber at resonance $\Omega \cong \omega_1$, is about 0.2, the system is stable with some chaos.

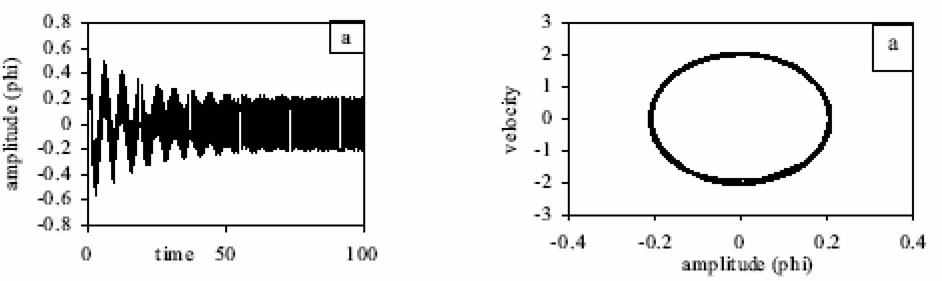


Fig. 5. System behaivor without absorber at primary resonance $\Omega \cong \omega_1$

Effects of the absorber: Fig. 6 illustrates the results when the absorber is effective, i.e., when $\Omega \cong \omega_1$, $\Omega \cong \omega_2$. It can be seen from the figure that for the main system the steady state amplitude is reduced to 50% of the maximum steady state amplitude and the oscillation is tuned. For the absorber, the steady state amplitude is about 9% of the excitation amplitude, with slight chaos. This means that the effectiveness of the absorber E_a (E_a= steady state amplitude of the main system without absorber / steady state amplitude of the main system with absorber) is about 2. Due to the complicated non-linearity of the system the ratio ω_2/ω_1 was varied to get the optimum working conditions.

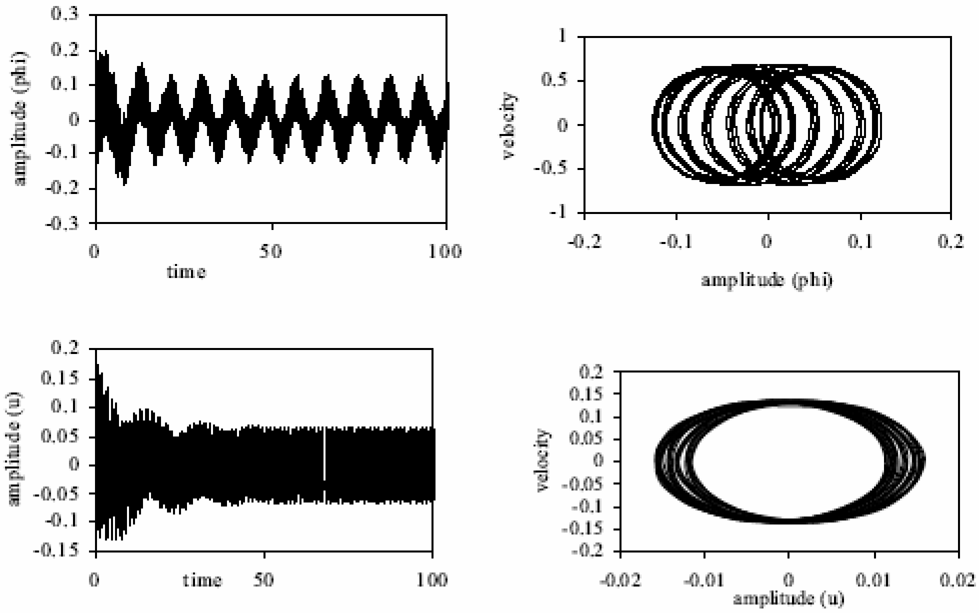


Fig. 6. Both system and absorber are in operation

$\Omega \cong \omega_1$, $\omega_2 \cong \omega_1$

The optimum working conditions were obtained when $\omega_2/\omega_1 \cong 0.74$ and $\Omega \cong \omega_1$, as shown in Fig. 7, where the steady state amplitude of the main system is reduced to less than 1% of the corresponding value shown in Fig. 5a, with increasing chaos. The effects on the absorber are trivial. From the engineering point of view the effectiveness of the absorber E_a is about 124. Figs. 8(a-b) illustrates the effects of variations of the natural frequency ω_2 on the steady state amplitude of the main system, for both considered resonance cases.

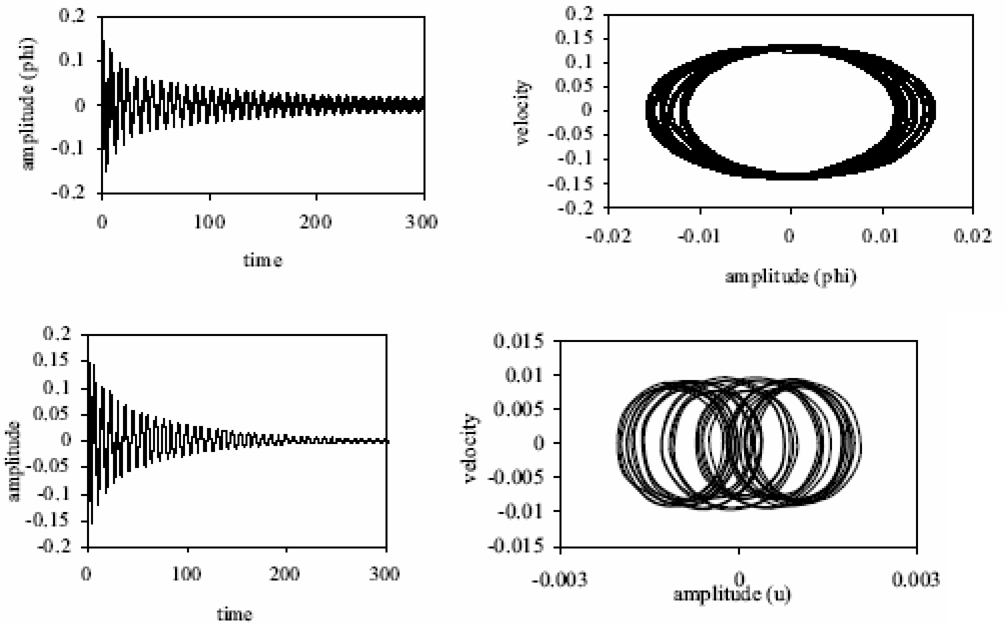


Fig. 7. Effects of the system when $\Omega \cong \omega_1$ $\omega_2 / \omega_1 \cong 0.74$

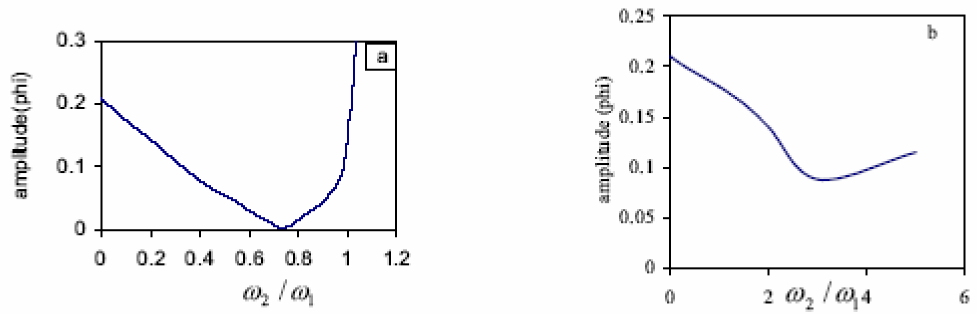


Fig. 8. The effects of variation in ω_2 $a)$ $\Omega \cong \omega_1$ $b) \Omega \cong \omega_2$

4.3 Active control

Active control is applied to improve the behavior of the simple pendulum at the primary resonance case $\Omega \cong \omega_1$, via negative velocity feedback or its square or cubic value. The equation of motion in this case is:

$$\ddot{\varphi} + \varepsilon \hat{c}_1 \dot{\varphi} + \omega_1^2 \sin(\varphi) + G\dot{\varphi}^n = f \cos \Omega t \qquad (50)$$

where G is the gain. Here, we are concerned with the effect of the gain G on the pendulum response. Three cases will be considered. They are when n=1, 2 and 3. Fig.9 shows a comparison between the three cases. It can be seen from the figure that for $G \leq 2.5$, the three cases are approximately the same. Also it can be seen from the figure that all three cases leads to saturation phenomena for large values of G. Comparing the effectiveness of the three methods we can see that:

a) For n =1, E_a =100 **b)** For n =2, E_a =13 **c)** For n =1, E_a =8

It is clear that best of them when n =1, this may be attributed to the fact that $\dot{\varphi} < 1$.

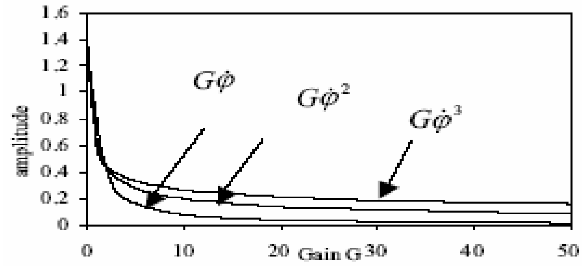


Fig. 9. The effects of the gain G

5. CONCLUSIONS

The vibrations of a non-linear simple pendulum having both quadratic and cubic non-linearties, subjected to harmonic excitation can be controlled applying either non-linear absorber (passive control) or negative velocity feedback or its square or cubic value (active control). This work is limited to single excitation force. Multiple time scale perturbation technique is applied to determine approximate closed form solutions for the coupled differential equations describing the system up to the second order approximation. Both frequency response equations and the phase-plane technique are applied to study system stability. The phase-plane is a good criterion for the presence of dynamic chaos. From the above study the following may be concluded.

1- The steady state amplitudes of the main system and absorber are monotonic increasing to the excitation amplitude.
2- The steady state amplitude of the main system is monotonic decreasing functions both natural frequency ω_1, and the damping coefficient c_1.
3- For passive control the effectiveness of the absorber is about E_a=124.
4- Optimum working conditions is not as the expected one, i.e. $\Omega \cong \omega_1$, $\omega_2 \cong \omega_1$, but they occur when $\Omega \cong \omega_1$and $\omega_2 / \omega_1 \cong 0.74$.
5- The vibration of the system can be controlled actively via negative velocity feedback, which can be used to reduce the amplitude of the system to about 1% of the system original value. This means that E_a =100. The gain value may be less than 8 for the considered cases, which is the gain value at saturation beginning.

REFERENCES

1. Mead, J. D., Passive Vibration Control, John Wiley & Sons, 1999.
2. M. Eissa, Vibration and chaos control in I. C engines subject to harmonic torque via non-linear absorbers, ISMV, Second International Symposium on Mechanical Vibrations. Islamabad, Pakistan, 2000.
3. M. Eissa, Vibration control of non-linear mechanical system via a neutralizer, Electronic Bulletin No 16, Faculty of Electronic Engineering Menouf, Egypt, July, 1999.
4. Cheng-Tang Lee *et al.*, Sub-harmonic vibration absorber for rotating machinery, ASME Journal of Vibration and Acoustics, 119, 590-595, 1997.
5. M. Eissa and El-Ganaini, Part I, Multi-absorbers for vibration control of non-linear structures to harmonic excitations, ISMV Conference, Islamabad, Pakistan, 2000.
6. M. Eissa and El-Ganaini, Part II, Multi-absorbers for vibration control of non-linear structures to harmonic excitations, ISMV Conference, Islamabad, Pakistan, 2000.

7. I. Y. Shen, Weili Guo and Y. C. Pao, Torsional vibration control of a shaft through active constrained layer damping treatments, Journal of Vibration and Acoustics, 119, 504-511, 1997.
8. N. Liu and K. W. Wang, A non-dimensional parametric study of enhanced active constrained layer damping treatments, Journal of Sound and Vibration, 223(4), 611-644, 1999.
9. R. Stanawy and D. Chantalakhana, Active constrained layer damping of clamped-clamped plate vibration, Journal of Sound and Vibration, 241(5), 755-777, 2001.
10. A. Baz, M. C. Ray and J. Oh., Active constrained layer damping of thin cylindrical shell, Journal of Sound and Vibration, 240(5), 921-935, 2001.
11. Y. M. Shi, Z. F. Li, X. H. Hua, Z. F. Fu and T. X. Liu, The modeling and vibration control of beams with active constrained layer, Journal of Sound and Vibration, 245(5), 785-800, 2001.
12. D. Sun and L. Tong, Modeling and vibration control of beams with partially de-bonded active constrained layer damping patch, Journal of Sound and Vibration, 252(3), 493-507, 2002.
13. K. Nagaya, A. Kurusu, S. Ikia, and Y. Shitani, Vibration control of structure by using a tunable absorber and optimal vibration absorber under auto-tuning control, Journal of Sound and Vibration, 228(4), 773-792, 1999.
14. N. Jalali, A new perspective for semi-automated structural vibration control, Journal of Sound and Vibration, 238(3), 481-494, 2000.
15. M. S. Tsai and K. W. Wang, On the structural damping characteristics of active piezoelectric actuators with passive shunt, Journal of Sound and Vibration, 221(1), 1-22, 1999.
16. A. J. Fleming and S. O. R. Moheimani, Optimization and implementation of multi-mode piezoelectric shunt damping systems, IEEE/ASME Transactions of Mechatronics, 7(1), 87-94, 2002.
17. S. Zhou and J. Shi, Active balancing and vibration control of rotating machinery: Asurvey, The Shock and Vibration Digest, 33(4), 361-371, 2001.