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COMPUTATIONAL ANALYSIS OF A MULTI-SERVER BULK ARRIVAL WITH TWO MODES SERVER BREAKDOWN

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Abstract- An extensive Monte Carlo study is performed to analyze a multi-server, bulk arrival ($M^{[x]}$/M/C; C-1/FCFS) queuing system. The system breakdown limits the system to serve with either C or (C-1) servers. The server breakdown has equal chance over all servers in the system while the arrival process is a Poisson process and the distribution of the bulk size X is a positive Poisson bulk size. Measures of system efficiency including mean queue length, mean waiting time, and blocking probability are introduced. Numerical results are obtained by simulation of the entire system.
Keywords - Bulk Arrival; Multi-server; Server breakdown; Queueing models.

1. INTRODUCTION

Many authors have investigated the analytical aspect of bulk-arrival queuing models through different techniques. Several authors presented the analytical solution for multi-server queuing systems with server breakdown. Very little seems to have been written about the numerical computations such as Alam, M. and Mani, V. [10] who analyzed the (M/M/1/N/FCFS) system. In their paper, they studied the (M/M/1/N/FCFS) system with a limited capacity N and two modes of operation for the server (either operative or non-operative), with different arrival and service rates for each mode under FCFS discipline. In their 1989 paper [11] they studied a multi-server, FCFS queuing system (M/M/S; S-1/N/FCFS) with S or S-1 servers available i.e. one server could be broken down. A recursive solution was used for computing steady state probabilities of such a system.

M.L. Chaudhry and W.K. Grassmann [12] analyzed a system ($M^{[x]}$ /M/C) in which arrivals occur in-bulks. Bulks of customers arrive at random times, and there is no limit on queue length. The system has C identical exponential servers in parallel. Service is on first come, first served basis. In their paper, they investigate specifically three distributions of the bulk size X, namely constant bulk sizes, positive Poisson bulk sizes and geometric bulk sizes. For these distributions, extensive numerical calculations have been done. This paper shows how to calculate fractiles for both the queue length and the waiting time distribution.

M.L. Chaudhry and G.Briere [6] studied numerically the bulk arrival queuing model ($M^{[x]}$/G/1). For a specified service-time distribution, an algorithm for the limiting distribution of the number in the system at random epoch is developed. This is also used to find the limiting distribution of the number in the system at pre-arrival and post-departure epochs.

M.L. Chaudhry, J.G.C. Templeton, and J.Medhi [8] studied finding the limiting distribution of the number in the system numerically for the bulk arrival, multi-server queuing system ($M^{[x]}$/D/C).

M.L. Chaudhry and Nam K. Kim [9] studied a complete and simple solution for the discrete time multi-server queue with bulk arrivals and deterministic service times.

Jau-Chanke. [5] studied two thresholds of an (M/G/1) queuing system with server breakdowns and two vacation types.

Shoukry, E.M., Gharraph, M.k., and Hassan, N.A. [2] analyzed a system (M/E_k /S; S-1/N/FCFS) with heterogeneous servers that operate in two modes. In addition, they [3] analyzed a system (M/E_k /S; S-1/N/FCFS) with homogeneous servers that operate in two modes.

In this paper a multi-server queuing model ($M^{[x]}$/M/C; C-1/FCFS) with homogeneous servers is discussed. This system has already been investigated by Chaudhry[8, 12] and Shoukry[2, 3]. The results of these authors are extended and corrected. This system has the following description. Customers arrive in batches according to compound Poisson process with mean batch arrival rate λ. Let N (t) be the number of batches of customers which have arrived by time t where {N (t), t ≥ 0} can be modeled as a Poisson process. Next, we fit a discrete distribution to the size of the successive batches, the size of arrival batch must be positive integers. Thus, for the original arrival process, it is assumed that batches of customers arrive in accordance with the arrival process {N (t), t ≥ 0}. The total number of customers that arrive by time t denoted by Z (t) is given by,

$$Z(t) = \sum_{i=1}^{N(t)} X_i \qquad for \qquad t \geq 0 \tag{1}$$

where x_i is the number of customers in the ith batch. The x_i's are assumed to be IID random variables independent of {N (t), t ≥ 0}, then the stochastic process {Z (t), t≥ 0} is said to be a compound Poisson process. In this case, the bulk size X has a positive Poisson distribution with no bulk size of zero. Now, let $\{a_x\}$ represent a probability sequence that governs bulk size. This probability a_x is defined as,

$$a_x = P(x;\theta) / [1-P(0;\theta)] = \frac{\theta^x e^{-\theta} / x!}{1-e^{-\theta}} \tag{2}$$

where θ is a parameter of bulk size. The average bulk size m in this case will be given by,

$$E(X) = \theta / (1-e^{-\theta}) = m \tag{3}$$

Customers are served according to exponential service time distribution with parameter $\mu > 0$. The C and C-1 in the model notation above are the number of parallel service channels. At any instant of time only one of the servers breaks down and the system operates with (C-1) servers instead of C servers who the system initially operates with. Thus a system alternates between two modes of a system operation. This is due to breakdown of one of the servers, to scheduling policy, or to one of the servers leaving the system temporarily. Service is on a first come, first served basis. In addition, the queue length in our model is assumed to have no limit.

The paper is organized in 7 sections: Section 2 presents a description of the study design. Section 3 introduces notations and basic assumptions for the model under study. Section 4 describes the methodology used for the analysis of the system together with code and routines used. Section 5 discusses an extensive Monte Carlo results obtained from analyzing the system. The results include different tables and graphs. Section 6 gives an example to show how the method works. Finally, section 7 concludes the paper.

The main intent of this paper is to present numerical results for certain measures of efficiency such as utilization, mean queue length, mean waiting time, average number in the system, the probability of no customer in the system and the blocking probability for this queuing system. Applications of such bulk-arrival, multi-server, and breakdown queuing model appear in different real life systems. Examples of such systems are bank, telecommunication control-facility, classic machine-repair problem, scheduling patients in hospital clinics, telephone exchanges, and taxi stands.

2. DESCRIPTION OF THE STUDY DESIGN

As indicated by the notation ($M^{[x]}$/M/C; C-1/FCFS), bulks of customers arrive at random times with mean bulk arrival rate λ**.** We investigate specifically a distribution of the bulk size, namely positive Poisson bulk sizes. The queuing system under study has homogeneous parallel servers where service time has exponential distribution with mean $1/\mu$**.** Our system alternate between two modes of system operation, this is due to breakdown of one of the servers. In one mode of operation all C servers are available and in the other mode only C-1 servers are available. The server is broken randomly. This mean that only one server is allowed to breakdown randomly according to a discrete uniform distribution that assigns one of the servers to be out of service. The mean time that the system operates with C servers and C-1 servers is $1/\alpha$ and $1/\beta$ respectively.

The queue discipline for bulks is FCFS while the service discipline within the bulks is based on randomly choosing one of the customers mentioned earlier. The conditional probability of the customer waiting for d departures before his service commences, given the state of the system n just before the arrival of the customer's bulk, is given by,

$$\sum_{r=c-n+d}^{\infty} \frac{ra_r}{m}\frac{1}{r} = \frac{1}{m}\sum_{r=c-n+d}^{\infty} a_r \qquad , d = 1, 2,\ldots$$

$$0 \le n \le d+C-1 \qquad (4)$$

where, the bulk size X is an r.v. with distribution given by a_r =P (X =r), r ≥1, X has mean m , $0 \le m = \sum_{r=1}^{\infty} ra_r < \infty$ and variance $\sigma_a^2, 0 < \sigma_a^2 < \infty$ (see [3]).

Customers of a certain bulk are served randomly. If X is the size of a bulk then customer ***i***, ***i*** =1,2,…,X has the same chance of joining service. In order to generate an equal chance to all customers in a given batch of size X, either a uniform assignment of the order in which they may be served is used or using a predefined permutation sequence. This permutation routine helps to recognize every customer in the batch.

3. NOTATION AND ASSUMPTIONS

In this section, the different notation and assumptions used in this article will be introduced.

3.1. Notation

C : Number of parallel servers.

λ : Mean bulk arrival rate.

μ : Constant service rate of a server.

m : The average bulk size.

L_Q : The average queue length.

W_S : The average waiting time per customer in the system.

L_S : The average number of customers in the system.

W_Q : The average waiting time per customer in the queue.

Mode-1: All C servers are available for serving the customers in this mode.

Mode-0: All (C-1) servers are available for serving the customers in this mode.

P_{0i} : The probability of having no customers in the system when the system is in mode-i, i= 0,1.

β : Transition rate from mode-0 to mode-1.

α : Transition rate from mode-1 to mode-0.

P_B : The blocking probability that defines the probability that all servers are busy.

ρ : The traffic intensity.

N : The number of batches.

θ : The parameter of the batch size distribution.

3.2. Assumptions

A1: The system at any instant of time is in one of two modes of operation mode-0 or mode-1.

A2: Groups of customers (bulks) arrive at a multi-server queuing system in accordance with Poisson process with parameter λ.

A3: The probability of two or more events occurring simultaneously is negligible.

A4: The queue has C identical exponential servers in parallel with each server having a service rate μ.

A5: The times that system operates with C servers and (C-1) servers have exponential distribution with mean 1/α and 1/β respectively.

A6: The bulk size X follows a positive Poisson distribution with parameter θ.

A7: The traffic intensity for the system is given by $\rho = m\lambda/C\mu$ and the condition for existence of a steady state solution is $\rho < 1$.

A9: There is no limit on queue length.

A10: Occasionally one of the C parallel servers breaks down and leaves the facility according to a discrete uniform distribution with parameter C.

A11: Bulks are served according to FCFS discipline while customers in a bulk are served randomly.

4. METHODOLOGY

The methodology can be described in the following steps:

1-Generate 10000 interarrival times for 10000 different batches from exponential distribution with mean $1/\lambda$.

2-Generate a random batch size for each batch in step 1 using positive Poisson distribution with parameter θ.

3-To determine the service times for each customer in the successive batches, generate random variables from exponential distribution with mean $1/\mu$ for each server.
4-Determine the intervals of time that the system operates with C servers by generating random variables from exponential distribution with mean $1/\alpha$.
5-To determine the intervals of time that the system operates with (C-1) servers, generate random variables from exponential distribution with mean $1/\beta$.
6- To determine which server will be down, generate random variables from discrete uniform distribution with parameter C. Where the integers 1, 2, 3,…,C occur with equal probability.
7- Calculate the event time of breakdown and repairing for each server based on the intervals of time during which the system works with C-1 and C servers respectively. (See step 4&5).
8- Calculate the traffic intensity ρ from equation $\rho = m\lambda/c\mu$.
9- Find the number of all customers that enter the system as a sum of sizes of batches that arrive to the system.
10- Find the arrival time of a batch.
11-Determin the mode at which an arriving batch will find the system as follows:
- If the arrival time of a batch is greater than the event time of breakdown and less than the event time of repairing, the system will be in mode-0.
- If the arrival time of a batch is greater than the event time of repairing and less than the event time of breakdown, the system will be in mode-1.

12-An arriving customer will immediately start service if one of servers is free or wait until any server becomes free.
13-Calculate the departure time of a customer from system as the sum of arrival time plus service time.
14-Calculate the waiting time of a customer in the queue.
15-Calculate the waiting time of a customer in the system.
16-Calculate the busy time for each server.
17- Repeat steps 12 to 16 until all customers in a given batch are served.
18-Calculate the departure time of a batch.
19-Calculate the sum of waiting times of customers in queue.
20-Calculate the sum of waiting times of customers in system.
21-If a server is broken down during serving a certain customer this customer will quit service and will start service at the first server available. As a result, repeat the steps 12 to 16 and 19 to 21 again.
22-Define the different time intervals during which system operates in the two modes.
23-Repeat the steps from 10 to 22, 10000 times.
24-Calculate the probability that the system is in mode-i , i= 0,1while there is no any customer in the system $P_{0,i}$ from relation,

$$P_{0,i} = \frac{\Sigma \text{ The number of batches that arrival to find the system is idle in mode-i, i=0,1}}{\text{The number of all batches}}$$

25-Calculate the probability that no customers are in system P_0 from relation,

$$P_0 = \frac{\Sigma \text{ The number of batches that arrival to find the system is idle}}{\text{The number of all batches}}$$

where, $p_0 = p_{0,0} + p_{0,1}$

26-Calculate the blocking probability P_B, i.e. the probability that any customer in an arriving batch must wait.

$$P_B = \frac{\text{The number of waiting customers in queue}}{\text{The number of all customers}}$$

27-Calculate the average waiting time in queue W_Q from relation,

$$W_Q = \frac{\Sigma \text{ the waiting time per customer in queue}}{\text{The number of all customers}}$$

28-Calculate the average waiting time in system W_S from relation,

$$W_S = \frac{\Sigma \text{ the waiting time per customer in system}}{\text{The number of all customers}}$$

29-Calculate the average queue length L_Q from relation,

$$L_Q = \frac{\Sigma \text{ the waiting time per customer in the queue}}{T}$$

30-Calculate the average number in the system L_S from relation,

$$L_S = \frac{\Sigma \text{ the waiting time per customer in system}}{T}$$

where T is the length of time we observed the system (which equals to the master clock time since it starts at zero).

5. RESULTS

The program was tested extensively for values of (ρ, C, X) with $0.1 \le \rho \le 0.9$, $1 \le C \le 100$ and batch size $X \le 100$ and it did not encounter any problems. We note that tables 1, 2, 3, 4, 5 and 6 contain two variables. The first variable represents the traffic intensity ρ and the second variable represents the relative transition rate $\alpha/(\alpha+\beta)$. While the different values inside each table represent performance measures (output data). These measures are $L_Q, L_S, W_Q, W_S, P_0, P_B$ in the respective tables 1- 6.

Based on the methodology explained in the previous section, the input data includes the number of servers C, the number of batches N, the parameter of size of batches θ, mean service time $1/\mu$, mean interarrival time $1/\lambda$, the mean time that the system operates with c servers $1/\alpha$ and the mean time that the system operates with (C-1) servers $1/\beta$.

Different performance measures are calculated. These measures include:

1-The expected number of customers in the queue and in the system be L_Q and L_S respectively.

2-The expected waiting time per customer in the queue and in the system are W_Q and W_S respectively.

3-The probability of having 0 customer in the system P_0.

4- The blocking probability P_B .

We note that the traffic intensity ρ changes with variation the value of $1/\mu$. So the value of relative transition rate α/(α+β) is varied due to the changes in values $1/\alpha$ and $1/\beta$. Results shown in table 1, 2, 3, 4, 5 and 6 are obtained by inserting these input data and running the program.

The graphs shown in figures 1, 2, 3, 4, 5, and 6 are illustration for tables 1, 2, 3, 4, 5, and 6 respectively. These graphs show the effect of performance measures on the system resulting for different traffic intensity ρ and relative transition rate α/(α+β).

For example, figure 1 indicates the relation between traffic intensity ρ and average number of customers in the queue L_Q with the increase of relative transition rate REL which equals α/(α+β).

Table 1
Average number in the queue
(Mean size of bulk=15.9907)
(Number of servers: 5)

ρ	α/(α+β)				
	0.000	0.250	0.500	0.750	1.000
.100	0.617	0.695	0.758	0.833	0.919
.200	1.646	1.843	2.047	2.295	2.528
.300	3.204	3.626	4.009	4.540	5.015
.400	5.395	6.222	6.963	7.975	9.154
.500	8.771	10.254	11.957	13.947	16.485
.600	14.598	17.201	20.649	25.076	32.756
.700	24.721	31.796	40.435	53.012	84.869
.800	48.370	64.949	101.905	185.776	480.074
.900	87.885	141.152	309.513	1199.145	5077.959

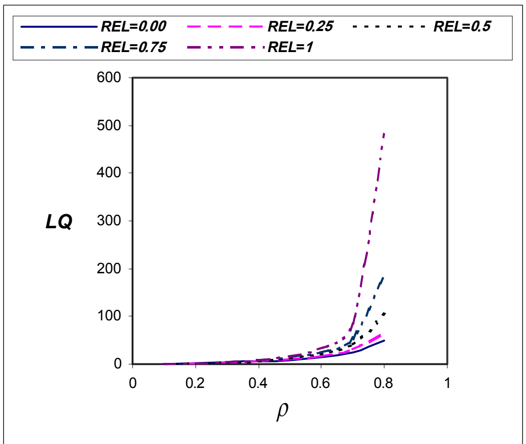

FIG. 1

Table 2
Average number in the system
Mean size of bulk=15.9907
(Number of servers: 5)

ρ	α/(α+β)				
	0.000	0.250	0.500	0.750	1.000
0.100	1.110	1.189	1.251	1.327	1.413
0.200	2.635	2.833	3.037	3.285	3.519
0.300	4.684	5.111	5.489	6.021	6.496
0.400	7.367	8.195	8.936	9.949	11.130
0.500	11.236	12.721	14.422	16.414	18.953
0.600	17.556	20.158	23.605	28.034	35.716
0.700	28.171	35.253	43.886	56.460	88.320
0.800	52.306	68.881	105.854	189.706	483.989
0.900	92.170	145.432	313.787	1203.364	5081.955

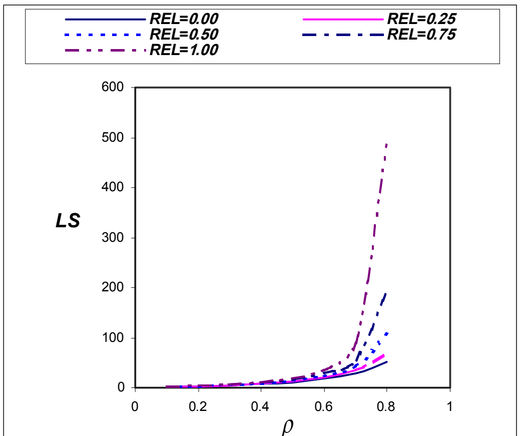

FIG. 2

Table 3
Average waiting time in the queue
(Mean size of bulk=15.9907)
(Number of servers: 5)

ρ	α/(α+β)				
	0.000	0.250	0.500	0.750	1.000
0.100	1.460	1.647	1.794	1.973	2.177
0.200	3.899	4.365	4.849	5.434	5.987
0.300	7.589	8.588	9.495	10.753	11.877
0.400	12.777	14.738	16.494	18.893	21.685
0.500	20.777	24.291	28.326	33.043	39.055
0.600	34.585	40.751	48.927	59.420	77.623
0.700	58.578	75.363	95.865	125.663	201.248
0.800	114.646	154.000	241.800	440.940	1143.611
0.900	208.349	334.808	736.470	2884.913	12917.871

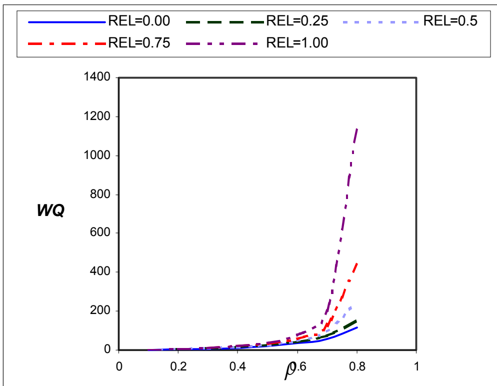

FIG. 3

Table 4
Average waiting time in the system
(Mean size of bulk=15.9907)
(Number of servers: 5)

ρ	α/(α+β)				
	0.000	0.250	0.500	0.750	1.000
0.100	2.628	2.816	2.963	3.142	3.347
0.200	6.241	6.709	7.192	7.779	8.334
0.300	11.092	12.103	13.000	14.260	15.385
0.400	17.447	19.733	21.167	23.569	26.364
0.500	26.616	30.135	34.166	38.887	44.901
0.600	41.591	47.757	55.932	66.429	84.638
0.700	66.752	83.556	104.047	133.836	209.432
0.800	123.976	163.323	251.168	450.269	1152.938
0.900	218.507	344.960	746.649	2895.062	12928.040

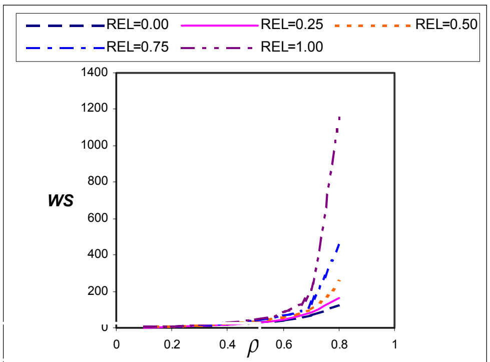

FIG. 4

Table 5
The probability that no customers are
in the system at the arrival batch
(Mean size of bulk=15.9907)
(Number of servers: 5)

ρ	α/(α+β)				
	0.000	0.250	0.500	0.750	1.000
0.100	0.831	0.824	0.818	0.815	0.806
0.200	0.697	0.687	0.675	0.662	0.655
0.300	0.585	0.562	0.551	0.532	0.519
0.400	0.476	0.456	0.437	0.415	0.399
0.500	0.382	0.352	0.338	0.309	0.289
0.600	0.293	0.264	0.239	0.218	0.189
0.700	0.215	0.178	0.155	0.130	0.096
0.800	0.148	0.109	0.076	0.047	0.014
0.900	0.0977	0.0554	0.0266	0.0011	0.0002

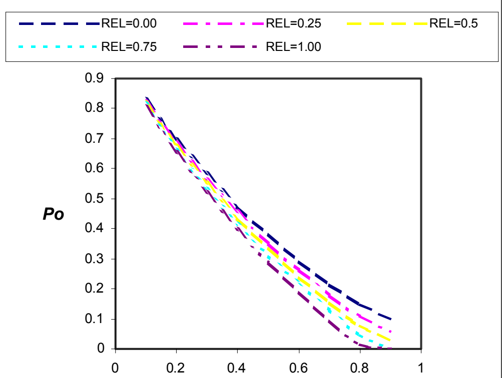

FIG. 5

ρ

Table 6
The probability that no servers are idle in the system
(The number of servers=5)

ρ	α/(α+β)				
	0.000	0.250	0.500	0.750	1.000
0.100	0.712	0.729	0.745	0.761	0.778
0.200	0.751	0.767	0.783	0.798	0.814
0.300	0.784	0.802	0.817	0.831	0.848
0.400	0.817	0.834	0.850	0.864	0.880
0.500	0.849	0.867	0.881	0.895	0.911
0.600	0.881	0.897	0.912	0.925	0.940
0.700	0.910	0.928	0.942	0.954	0.969
0.800	0.938	0.955	0.970	0.983	0.995
0.900	0.9579	0.9761	0.9894	0.9996	0.9999

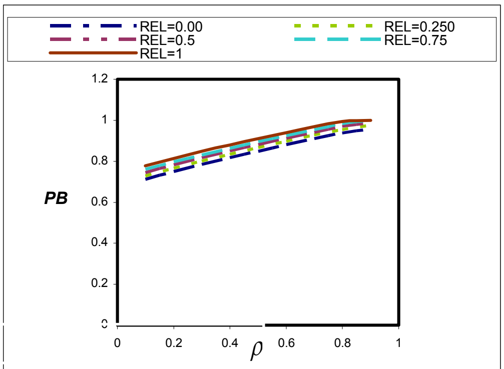

FIG. 6

6. EXAMPLE

This example considers a system that operates initially with three homogenous servers. When one of the servers breaks down, the system operates with two homogenous servers (system is in mode-0). After the repair of the broken server and putting it back into service the system re-operates with three homogenous servers again. Hence, the system alternates between two modes of system operation. There is no limit on system capacity.

Now, consider five bulks that arrive at times 0.344, 13.787, 16.449, 22.082, and 23.768. The sizes of bulks are 8, 3, 6, 4, and 6. Thus the three servers are to serve 27 customers that arrive in bulks according to exponential distribution with means 2.985. Service times for each server are 0.103, 4.033, 0.799, 0.506, 9.353, 0.039, 1.526, 0.328, and 2.896. The servers are broken down based on discrete uniform distribution. The order of which servers break down is server 3, server 1, server 3 and finally server 2. The time intervals that the system operates with three servers have an exponential distribution with mean 5.190. These times are 3.302, 1.001, 3.777, and 4.957. The time intervals that the system operates with two servers have an exponential distribution with mean 15.567. These times are 9.905, 3.026, 11.331, and 14.872. The first bulk consists of 8 customers of which 3 customers start the service immediately without waiting and take service time equals 0.103 for each one of them then depart the system at 0.447. The remaining 5 customers of the first bulk will be waiting until any server becomes available. At time 0.447 all servers are available. So 3 customers will enter the service and each one of them takes service time equals 4.033 and departs at 4.480 except customer number 6. During serving customer 6 at server 3, this server breaks down at time equals 3.302. Now, the system operates with server 1 and server 2 (i.e. the system is in mode-0). The customer number 6 will start the service again at the first server available. The server 1 and server 2 are ready for serving at time 4.480. So the customer 6 and customer 7 will start the service at time 4.480 and each one of them takes service time equals 0.799 then depart at time 5.278. At time equals 13.207, the server 3 will put back into the service in the system (i.e. the system is in mode-1). Other customers in the system and new arriving batches will be similarly served. We note that the last bulk will depart the system at time equals 43.792. The corresponding performance measures for this system will be L_Q= 2.188, L_S= 3.876, W_Q= 3.726, W_S=6.601, P_0=0.400, and P_B= 0.741.

7. CONCLUSION

Based on the previous results, this investigation can be concluded by: -

1- Tables 1 and 2 show that the increase of relative transition α/(α+β) has remarkable effect on the average number of customers in the queue and in the system. While the increase of the traffic intensity ρ has a noticeable effect on the same.

2- The average number of customers in the queue and in the system increase as the average bulk size increases. So the increase of relative transition α/(α+β) has noticeable effect on L_Q and L_S when the average bulk size increases.

3- The most effect on the average number of customers in the system and in the queue is happening when the traffic intensity ρ is very close to unity. The absence of one server affects on the average number of customers as traffic intensity near the unity (heavy traffic).

4- Tables 3 and 4 show that the average waiting time in the queue and in the system are highly affected by the absence of one of the servers for both low and high traffic intensity.

5- Table 5 shows that the probability of having no customers in the system p_0 when a batch arrives i.e. the percentage of time the system is idle is not highly affected by the absence of one of the servers. While it decreases as the relative transition α/(α+β) increases for low traffic intensity and decreases noticeably as the relative transition α/(α+β) increases for high traffic intensity. Also we note that it decreases as the traffic intensity increases.

6-Table 6 shows that the blocking probability P_B is highly affected by the increase of traffic intensity. While it increases as the traffic intensity increases. So the blocking probability has remarkable effect resulting from absence of one of the servers. Where it increases as the relative transition α/(α+β) increases.

Finally, this paper was to come up with easily implementable algorithms to solve problems involving bulk-arrival queues with a breakdown of one server in case of steady state. This approach was preferred to producing large tables of exact results, varying the queuing parameters because of the endless list of possible combinations when applied to bulk queues and one of the servers breaks down. The performance measures are changed in response to the changes on the operating parameters. We documented the behavior of the system when one of the servers temporarily leaves the system with useful graphical representation to give the reader an opportunity to watch the system behavior over the traffic intensity and the relative transition rate.

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