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DISCRETE SINGULAR CONVOLUTION FOR FREE VIBRATION ANALYSIS ANNULAR MEMBRANES

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Abstract- In this paper, we investigate the effects of some geometric parameters and density variation on frequency characteristics of the annular membranes with varying density. The discrete singular convolution method based on regularized Shannon's delta kernel is applied to obtain the frequency parameter. The obtained results have been compared with the analytical and numerical results of other researchers, which showed well agreement.

Key Words- Free vibration, discrete singular convolution, annular membrane.

1. INTRODUCTION

Membrane structures are frequently encountered in most practical acoustical and technological applications. Free vibration analysis of such structures is the most important task for engineer in the design stage of microphones, pumps, pressure regulators, and other acoustical applications. Analytical and numerical studies of the free vibration of circular and annular membranes have also received a good deal of attention. Hence, many researches in this area have been carried out. Free vibration analysis of annular and circular membrane has been solved by several authors [1-7]. An analysis of the free vibration of circular and annular membranes has been presented by Laura et al. [1]. Jabareen and Eisenberger [2] proposed an exact method for free vibration analysis of non-homogeneous circular and annular membranes. Buchanan and Peddieson [3,4] and Buchanan [5] used Ritz and finite element method respectively, for vibration analysis of circular and elliptic membranes with variable density. Exact power series solutions for axisymmetric vibrations of circular and annular membranes with continuously varying density were presented by Willatzen [6]. Differential quadrature method was used for free vibration analysis of circular membrane by Gutierrez et al. [7].

The aim of the present paper is to present the DSC method for free vibration analysis of non-homogeneous annular membranes. We examine the discrete singular convolution method for free vibration problem of annular membranes. The performance of the method is tested for free vibration analysis of membranes considering a number of problems. The results are compared, wherever possible, with the available analytical and numerical solutions. This is the first instance in which the DSC method has been adopted for free vibration analysis of annular membranes.

2. DISCRETE SINGULAR CONVOLUTION

The discrete singular convolution (DSC) method is an efficient and useful approach for the numerical solutions of differential equations. This method introduced by Wei [8]. In the present paper, details of the DSC method are not given; interested readers may refer to the works of [9-16]. Since it was first introduced by Wei [10], the discrete singular convolution method had been applied to many problems [17-23]. Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by [11]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \,, \tag{1}$$

where $T(t-x)$ is a singular kernel. For example, singular kernels of delta type [12]

$$T(x) = \delta^{(n)}(x)\,; \quad (\text{n} = 0,1,2,...,). \tag{2}$$

Kernel $T(x) = \delta(x)$ is important for interpolation of surfaces and curves, and $T(x) = \delta^{(n)}(x)$ for n>1 are essential for numerically solving differential equations. With a sufficiently smooth approximation, it is more effective to consider a discrete singular convolution [13]

$$F_\alpha(t) = \sum_k T_\alpha(t - x_k) f(x_k)\,, \tag{3}$$

where F_α (t) is an approximation to $F(t)$ and $\{x_k\}$is an appropriate set of discrete points on which the DSC (3) is well defined. Note that, the original test function $\eta(x)$ has been replaced by f(x). Recently, the use of some new kernels and regularizer such as delta regularizer [14-21] was proposed to solve applied mechanics problem. The Shannon's kernel is regularized as [16]

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right];\ \sigma>0. \tag{4}$$

where Δ is the grid spacing, σ is a regularization parameter. It is also known that the truncation error is very small due to the use of the Gaussian regularizer, the above formulation given by Eq. (4) is practical and has an essentially compact support for numerical interpolation. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ given by [17]

$$\left.\frac{d^n f(x)}{d x^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i - x_k) f(x_k); \quad (n=0,1,2,...,). \tag{5}$$

where $\delta_{\Delta}(x - x_k) = \Delta \delta_{\alpha}(x - x_k)$ and superscript (n) denotes the n*th*-order derivative, and 2M+1 is the computational bandwidth which is centered around x and is usually smaller than the whole computational domain. For example the second order derivative at x=x_i of the DSC kernels is directly given by [18]

$$f^{(2)}(x) = \left.\frac{d^2 f}{d x^2}\right|_{x=x_i} \approx \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x_N) f_{i+k,j}. \tag{6}$$

Second-order derivative in Eq. (6) is given as [19];

$$\delta^{(2)}_{\pi/\Delta,\sigma}(x_m - x_k) = -\frac{(\pi/\Delta)\sin(\pi/\Delta)(x - x_k)}{(x - x_k)} \exp[-(x - x_k)^2/2\sigma^2]$$

$$-2\frac{\cos(\pi/\Delta)(x - x_k)}{(x - x_k)^2} \exp[-(x - x_k)^2/2\sigma^2)]$$

$$-2\frac{\cos(\pi/\Delta)(x - x_k)}{\sigma^2} \exp[-(x - x_k)^2/2\sigma^2] + 2\frac{\sin(\pi/\Delta)(x - x_k)}{\pi(x - x_k)^3/\Delta} \exp[-(x - x_k)^2/2\sigma^2)]$$

$$+\frac{\sin(\pi/\Delta)(x - x_k)}{\pi(x - x_k)\sigma^2/\Delta} \exp[-(x - x_k)^2/2\sigma^2] + \frac{\sin(\pi/\Delta)(x - x_k)}{\pi\sigma^4/\Delta}(x - x_k)\exp[-(x - x_k)^2/2\sigma^2)] \tag{7}$$

3. GOVERNING EQUATIONS

Consider an annular membrane of outer radius b, inner radius a and the radial coordinate r. The dimensionless governing differential equation for free vibration can be given as [1]

$$r\frac{\partial^2 W}{\partial r^2} + \frac{\partial W}{\partial r} + \Omega^2 f(r) r W = 0, \tag{8}$$

Where W is the transverse deflection, ρ is the mass per unit area, ω is the circular frequency, and T is the tension per unit length. The density of the membrane is the linear function of the x and given in non-dimensional form written as follows:

$$\rho(r) = \rho_0[1+\alpha(r)]$$

In order to simplify, we introduce the following dimensionless quantities:

$$r = a/b,\ \Omega^2 = \omega^2 b^2 \rho_0 / T \tag{9}$$

Applying the discrete singular convolution to the governing equation yields

$$r\sum_{j=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta r)W_{i,j} + \sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta r)W_{i,j} + \Omega^2 f(r)r_i W_i = 0, \tag{10}$$

The boundary conditions are as follows:

$$W=0 \qquad \text{at edges} \tag{11}$$

In the present study, we can't obtain reliable results for standard grid distributions. In this study we use the below formula for grid points in radial directions as proposed by Wang and Wang [24]

$$r_i = b + \frac{a-b}{2}\left[\frac{i-1}{N-1}\right] \tag{12}$$

4. NUMERICAL RESULTS

To validate the accuracy and applicability of the present formulation, the numerical results for annular membrane are compared to the results of Gutierrez [7] by the Ritz, differential quadrature and the exact solution. The obtained frequency values are given in Table 1 for different radius ratio of the circular (a/b=0) and annular membranes. It is very clear from Table 1 that the rate of convergence is very good for both annular and circular membranes with the increase in the grid numbers. It is also seen in this table that the present method yields accurate results.

Fig.1 shows the variation of fundamental frequency versus a/b for different densities. In general, the frequency increase with an increase in the radius ratio for membranes with different values of α. Fig.2 shows the effect of inhomogeneity parameter α on frequency for different radius ratio. It is concluded that the frequency parameter generally increases as radius ratio increase. It may be also noticed that with increasing density parameter, the frequency decreases.

Table 1. Fundamental frequency of annular homogeneous membranes

Methods	a/b=0.4	a/b=0.6	a/b=0.8
Ref. 7 (Exact)	5.1831	7.8284	15.6981
Ref.7 (DQ)	5.1830	7.8284	15.6981
Ref.7 (FEM)	5.1867	7.8337	15.7085
Present DSC Results N=M=11	5.1902	7.8305	15.7003
Present DSC Results N=M =13	5.1856	7.8288	15.6986
Present DSC Results N=M =15	5.1833	7.8285	15.6981
Present DSC Results N=M =17	5.1830	7.8285	15.6981

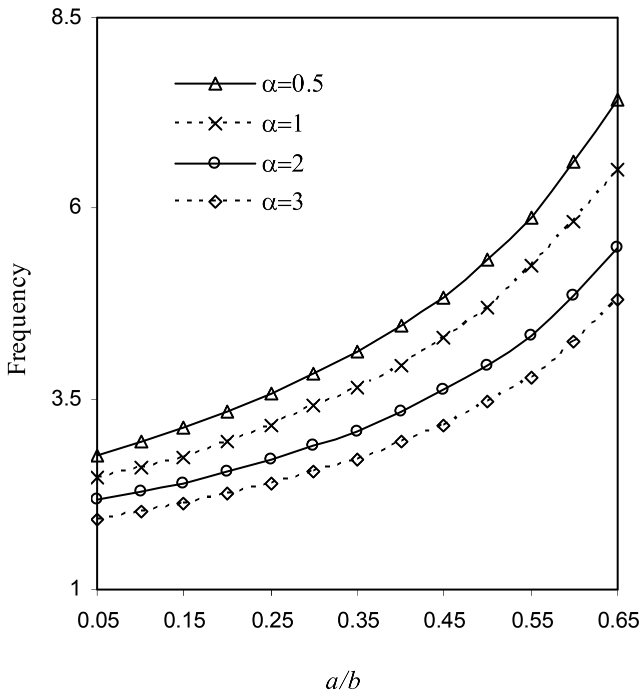

Fig. 1. Variation of fundamental frequency versus *a/b* for different densities

Fig. 3 displays the effects of inner-to-outer radius ratio on the frequency value. Linear density is considered. It is clear that the inner-to-outer radius ratio is an effective magnitude on frequency value. The frequencies for the annular membrane increase quickly with inner-to-outer radius ratio at any mode number. This increase in the frequencies with the *a*/*b* ratio is due to an increase in the inner radius of the annulus.

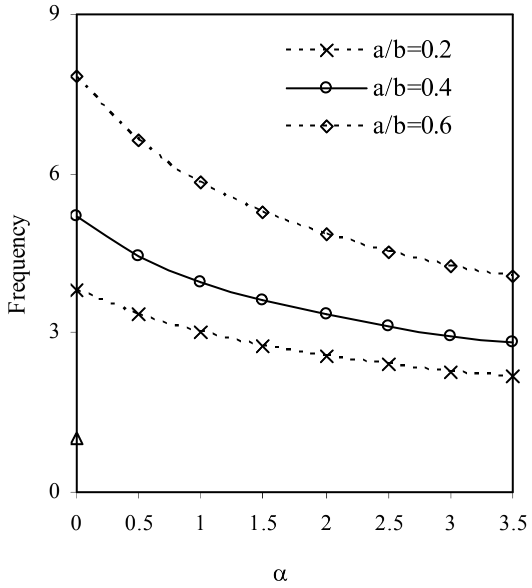

Fig. 2. Variation of fundamental frequency of annular membrane versus α for different values of *a/b*

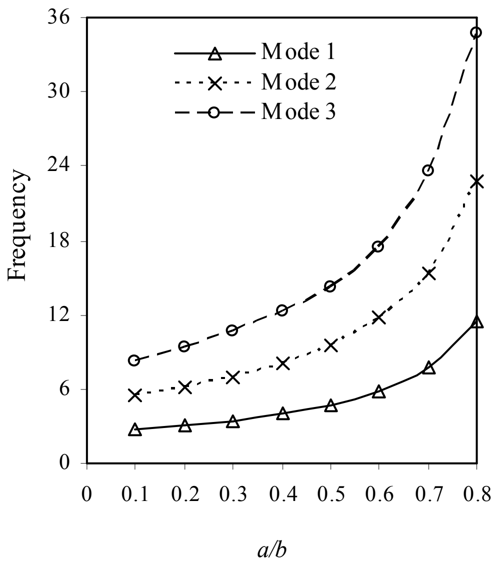

Fig. 3. Variation of first three frequency values of annular membrane versus *a/b* for linear density ($\alpha = 1$)

5. CONCLUSIONS

In the present paper, the discrete singular convolution method is applied to free vibration problem for annular membranes with varying density. The effects played by inner-to-outer radius ratio, variation of density, and mode number are studied. Numerical examples illustrating the accuracy and convergence of the DSC method for free vibration problem of annular membranes are presented. It is found that the convergence of the DSC approach is very good and the results agree well with those obtained by other researchers.

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