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DESIGN OPTIMIZATION OF ELECTRIC MOTORS BY MULTIOBJECTIVE FUZZY GENETIC ALGORITHMS

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Abstract-This paper presents a multiobjective fuzzy genetic algorithm optimization approach to design the submersible induction motor with two objective functions: the full load torque and the manufacturing cost. A multiobjective fuzzy optimization problem is formulated and solved using a genetic algorithm. The optimally designed motor is compared with an industrial motor having the same ratings. The results of optimal design show the reduction in the manufacturing cost, and the improvement in the full load torque of the motor.

Key Words- Multiobjective Fuzzy Optimization, Submersible Induction motor, Genetic Algorithms

1. INTRODUCTION

In recent years, Genetic Algorithm (GA) has been used as potent tools in design optimization of electrical machinery[1,2]. Unlike the standard Non-Linear Programming techniques (NLP), the GAs are able to find the global minimum instead of a local minimum. It does not require the use of the derivative of the function. When dealing with real measurements involving noisy data, the derivative of function is not always easily obtainable or may not even exist [3].

Optimization of electric machines should be realized by making trade-off between different objectives. For example, size of the machine should be small, it should be inexpensive, its efficiency and power factor should be good, etc. Taking note of these, the importance of multiobjective optimization is understood in this field. Several multiobjective approaches for the electrical machines design have been proposed. Kim et al [4] presented the multiobjective optimal design of induction motor for electric vehicle using a modified evolution strategy, and used the weighting method among the multiobjective optimization methods. Mirzaeian et al [5] proposed a novel multiobjective method for optimal design of a switched reluctance motor with two objective functions.

A previous study shows the application of GA technique to optimize the 75hp three-phase induction motors. The objective functions are separately optimized by using GA[6]. In this paper, an approach for multiobjective design optimization of submersible induction motor with two objective functions, i.e., the manufacturing cost and the full load torque utilizing the concept of fuzzy sets, convex fuzzy decision-making and a genetic algorithm having feature of a unique search [7] are presented. In the method, the objective functions are combined by fuzzy memberships so that the chromosomes with best performances for all objective functions have more chances to be chosen for participation in the next generation. The multiobjective fuzzy genetic algorithm (MFGA) optimization results show that the proposed approach is viable and reliable.

2. MULTIOBJECTIVE FUZZY OPTIMIZATION

A multiobjective optimization problem can be defined as follows[11]:
Find X which
Minimize f(X),
Subject to $h_i(X)=0$, $i=1,2,.....,m,$

$$g_j(X) \le 0, \quad j=1,2,\ldots,J, \tag{1}$$

$$X_k^u \ge X_k \ge X_k^l, \quad k=1,2,\ldots,K,$$

where $f(x)=[f_1(X), f_2(X),\ldots\ldots,f_n(X)]$ is a vector objective function, $h_i(X)$ and $g_j(X)$ are equality and inequality constraint functions, respectively. X_k^u and X_k^l are the upper and lower bounds of X_k, respectively.

It needs the membership function of each fuzzy objective function. Most applications that involve fuzzy set theory tend to be independent of the specific shape of the membership functions [12]. The fuzzy objective function can be stated using the following membership function representation [11].

$$\mu_{fi}(X) = \begin{cases} 1 & \text{if } f_i(X) \le f_i^{\min} \\ 1-2\left(\dfrac{f_i(X)-f_i^{\min}}{f_i^{\max}-f_i^{\min}}\right)^2 & \text{if } f_i^{\min} < f_i(X) \le f_i^{av} \\ 2\left(\dfrac{f_i(X)-f_i^{\max}}{f_i^{\max}-f_i^{\min}}\right)^2 & \text{if } f_i^{av} < f_i(X) \le f_i^{\max} \\ 0 & \text{if } f_i(X) > f_i^{\max} \end{cases} \tag{2}$$

where $\mu_{fi}(X): \Re^n \to [0,1]$ and $\mu_{fi}(X)$ measures the degree of satisfaction for any $X \in \Re^n$ in *i*th fuzzy objective function. $f_i^{\max}$, f_i^{av} and $f_i^{\min}$ are the maximum feasible value, average feasible value and minimum feasible value of *i*th objective function, respectively and are defined as:

$$f_i^{\min} = \min_i f_i(X^*), \quad f_i^{\max} = \max_i f_i(X^*) \quad \text{and} \quad f_i^{av} = \frac{f_i^{\min} + f_i^{\max}}{2} \tag{3}$$

where X^* is the solution for each of the objective functions.
The fuzzy constraints membership function is defined as follows [11]:

$$\mu_{gj}(X) = \begin{cases} 0 & \text{if } g_j(X) > b_j + d_j \\ 1-\left(\dfrac{g_j(X)-b_j}{d_j}\right) & \text{if } b_j \le g_j(X) \le b_j + d_j \\ 1 & \text{if } g_j(X) < b_j \end{cases} \tag{4}$$

where $\mu_{gj}(X): \Re^n \to [0,1]$ and $\mu_{gj}(X)$ measures the degree of satisfaction for any $X \in \Re^n$ in *j*th fuzzy constraint. The degree of satisfaction of *j*th constraint varies in between 0 and 1. For each fuzzy constraint, the allowable tolerances are given by d_j.

Fuzzy decision-making

Using membership function the objective functions and constraints are defined as fuzzy subsets. The optimal decision is implemented by selecting the best alternative from the fuzzy decision space (D). This is given as follows:[11]

$$\mu_D(X^*) = \max \mu_D(X) \tag{5}$$

where $\mu_D \in [0,1]$. One of the three fuzzy decisions can selected: intersection decision, convex decision and product decision. In the present study Convex decision-making principles are utilized. The convex decision [13] providing a framework to incorporate the relative importance of all the objectives and constraints uses the arithmetic mean. This can be defined as follows [11]:

$$D = \alpha f(X) + \beta g(X) \tag{6}$$

where α and β are weighting factors, which satisfy

$$\alpha + \beta = 1 \quad \alpha \geq 0 \quad \beta \geq 0$$

The weights α_i and β_j can be obtained from a linear weighted sum as follows;

$$\alpha_i = \frac{\mu_{fi}}{\sum_{i=1}^{n} \mu_{fi}} \quad \text{and} \quad \beta_j = \frac{\mu_{fj}}{\sum_{j=1}^{m} \mu_{fj}} \tag{7}$$

Thus for the convex decision the membership function can be stated as:

$$\mu_D(X) = \sum_{i=1}^{n} \alpha_i \mu_{fi} + \sum_{j=1}^{m} \beta_j \mu_{gj} \tag{8}$$

where α_i and β_j satisfy

$$\sum_{i=1}^{n} \alpha_i + \sum_{j=1}^{m} \beta_j = 1$$
$$\alpha_i \geq 0 \quad i = 1,2,.....,n \tag{9}$$
$$\beta_i \geq 0 \quad j = 1,2,.....,m$$

The multiobjective fuzzy optimization problem can be converted into single-objective optimization problems as follows:

$$\max \mu_D(X) = \sum_{i=1}^{n} \alpha_i \mu_{fi} + \sum_{j=1}^{m} \beta_j \mu_{gj} \tag{10}$$

Subject to

$$g_i(X) \leq b_j + d_j \qquad j = 1,2,......m$$

3. IMPLEMENTATION OF THE OPTIMAL DESIGN PROCEDURE

A submersible induction motor, having specifications shown in Table 1, was chosen for optimization. The motor's equivalent circuit model shown in Fig. 1 was used. This model is basically a per phase representation of a balanced poly-phase induction machine in the frequency domain [9, 10]. The objective functions have to be defined to evaluate each motor design. In this study, the manufacturing cost and full load torque are selected as the objective function.

Table 1. Specifications of submersible induction motor

Type	8 inc
Number of phases	3
Rated line voltage	380V
Number of poles	2
Connection	Δ
Power	75 hp
Frequency	50 Hz
Synchronous speed	3000 rpm

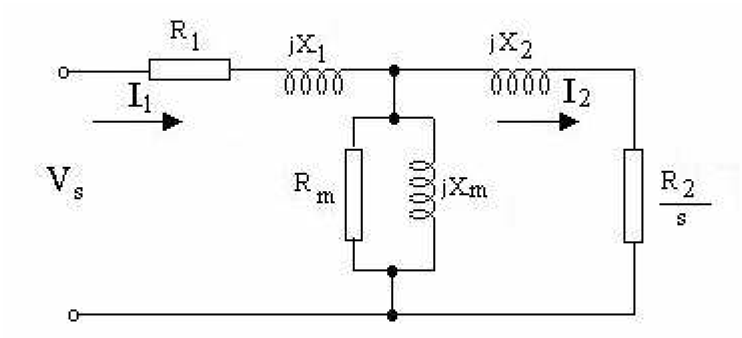


Figure 1. The per-phase equivalent circuit model of a submersible induction motor

First objective function: The cost variable consists of the laminations cost, copper cost, rotor-end-ring cost, and the core punching cost, which are used as the objective function of the optimization [10].

The weight of iron, W_{Fe}, is

$$W_{Fe} = \frac{L_1 SF D_o^2 P_{fe}}{4} \tag{11}$$

the weight of the stator winding, W_{sw}, is

$$W_{sw} = L_1 S_1 A_{1m} f_{ew} P_{sw} \tag{12}$$

and the weight of the rotor conductors, W_{rw}, is

$$W_{rw} = P_{rw}\left\{\frac{\pi w_a (D_r^2 - (D_r - 2w_r)^2}{2} + S_2 A_b (L_2 - 2w_a)\right\} \tag{13}$$

The punching cost C_p is estimated as 20% of the total cost. Thus the total cost or also the objective function is expressed as follows:

$$C_{total} = W_{Fe} Fe_{cost} + (W_{sw} + W_{rw}) Cu_{cost} + C_p \tag{14}$$

Second objective function: It should be noted that the full load torque as second the objective function is defined as:

$$T_n = \frac{60}{2\pi n_1} m \frac{V_s^2}{\left(R_1 + \tau_1 \frac{R_2}{s}\right)^2 + (X_1 + \tau_1 X_2)^2} \frac{R_2}{s} \tag{15}$$

where the τ_1 is the correction factor, the n_1 is the synchronous speed.

As shown in eqn (14) and eqn (15), it has been already selected two objective functions. Accordingly, the membership functions of manufacturing cost and full load torque for the submersible induction motor corresponding to the fuzzy objective functions are defined as:

$$\mu_C = \begin{cases} 1 & \text{if } C < 693 \\ 1 - 2\left(\dfrac{C-693}{945-693}\right)^2 & \text{if } 693 \le C \le 819 \\ 2\left(\dfrac{C-945}{945-693}\right)^2 & \text{if } 819 < C \le 945 \\ 0 & \text{if } C > 945 \end{cases} \tag{16}$$

$$\mu_{Tn} = \begin{cases} 0 & \text{if } T_n < 25 \\ 2\left(\dfrac{T_n-25}{200-25}\right)^2 & \text{if } 25 < T_n \le 112.5 \\ 1 - 2\left(\dfrac{T_n-200}{200-25}\right)^2 & \text{if } 112.5 < T_n \le 200 \\ 1 & \text{if } T_n > 200 \end{cases} \tag{17}$$

In the present study, it is assumed that fuzzy constraints ($\beta_j = 0$) is zero in eqn (10). Then, the multiobjective fuzzy optimization is defined as:

Which maximize $\mu_D = \alpha_1\mu_C + \alpha_2\mu_{Tn}$ (18)

Subject to

Rated slip, $s \le 0.06$

Stator yoke flux density, $B_{sy} \le 1.6\,T$

Rotor yoke flux density, $B_{ry} \le 1.6\,T$

Stator teeth flux density, $B_{st} \le 2\,T$

Stator slot filling factor, $F_f \le 0.2$

Starting current to a rated current ratio, $I_{start}/I_n \le 4.5$

Power factor, $\mathrm{Cos}\varphi \ge 0.79$

Pull-out torque to a rated torque ratio, $T_p/T_n \ge 1.9$

In this point, the Genetic algorithms (GAs) were used to solve this problem. Table 2 shows the practicable domains and the resolution for the design parameters. Here, it is assumed that the stator exterior diameter for the submersible induction motor is fixing. In the proposed design, each individual is classified by a string of 100 digits, composed by ten substrings, each of them representing an independent variable as reported in Table 2. The flowchart of the design optimization procedure for the MFGA optimization is depicted in Fig.2. An explanation of each step is given in the following:

i) Initial

Execution of the program starts with the performance specifications such as the initial motor design variables. The initial population is randomly generated, each of length *l*. Each individual represents a possible solution to the problem. The stator and rotor parameters are separately calculated.

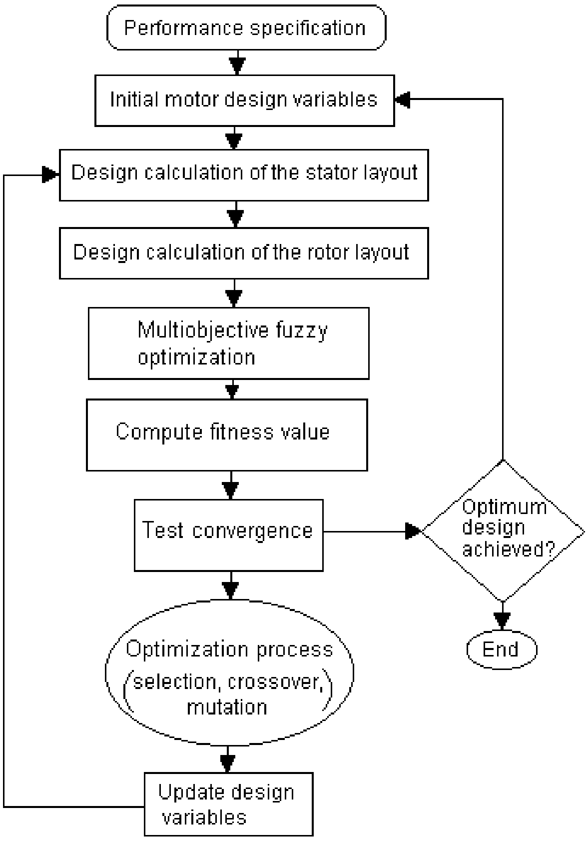


Figure 2. The flowchart of the design procedure for MFGA optimization

Table 2. Design parameters and their limit values

Description		Lower limit	Upper limit	Number of bits
N_1	Number of turns per phase	28	44	12
L_1	Stator iron length (cm)	65	80	12
w_a	End-ring width (mm)	10	30	12
D_i	Stator interior diameter (cm)	8	10.5	12
h_1	Stator slot height (mm)	15	25	8
w_1	Stator slot width (mm)	10	15	8
g	Air-gap (cm)	0.04	0.08	12
e	Bridge thickness of rotor closed slot (cm)	0.03	0.065	8
r_b	Rotor bar diameter (mm)	7.5	10	8
D_o	Stator exterior diameter (cm)	18	18	8

ii) *Multiobjective fuzzy optimization function*

To compute the degree of satisfaction of objective function and to check whether any of the constraints exceeded, the design calculations are transferred into the multiobjective fuzzy optimization. The degree of satisfaction values of each individual is transferred for evaluating in the fitness function.

iii) Fitness function

The performances of population members are evaluated by using fitness function. Then the fitness values of each individual can be calculated as follows:

$$\mu'_{Di} = \begin{cases} \mu_{Di} - P(g_j(x)) & \text{If} \quad \mu_{Di} - P(x) > 0 \\ 0 & \text{If} \quad \mu_{Di} - P(x) \le 0 \end{cases} \tag{19}$$

where μ_{Di} is the satisfaction degree of *i.* the individual, and $P(x)$ is the penalty function. The constrained problems are converted to unconstrained problems By means of penalty function. According to the constraints, penalty function is defined as following:

$$P(x) = \frac{1}{r} C_N \mu_{Di} \tag{20}$$

Where r is the number of total constraints, C_N is the number of constraint exceeded.

In the Eqn. (19), it is worth noting that the penalty function becomes inactive when the constraint inequality is satisfactory, and that the penalty function becomes active when the constraint inequality is unsatisfied.

iv) Selection strategy

The selection of parents to produce successive generations plays an important role in the GA. The goal is to allow the fittest individuals to be selected to reproduce process. In this work, the roulette wheel selection strategy is used.

v) Crossover operation

Crossover is a mixing operator that combines genetic material from selected parents. In this case, single-point crossover is used to perform crossover operation.

vi) Mutation operation

This is a common genetic manipulation operator, and it involves, the random alteration of genes during the process of copying a chromosome from one generation to the next. Raising the ratio of mutations increases the algorithm's freedom to search outside of the current region of parameter space. Mutation changes from a "1" to a "0" or vice versa. In this case, bit mutation, and elitism is used.

vii) Reinsert offspring

At this point a new population is achieved: all individuals are evaluated as described in step 2 and the subsequent steps are repeated. The procedure is stopped ('test convergence' in Fig.2) after a prefixed number of generations.

4. THE RESULTS

The MFGA optimization procedure has been successfully applied to optimize the manufacturing cost and the full load torque of submersible induction motor. The performance results of submersible induction motor obtained from MFGA optimization are found to be quite satisfactory. Particular attention has to be given to choice of the GA parameters. The population size greatly affects the quality of the result and computation time. It has been observed that small populations exhibit large fluctuations both in the average and best fitness while great populations cause premature convergence. However, while with a high P_c produces many new strings in the new population, deteriorating the successive generations, on the other hand, a low P_c causes a contracted search that can be ineffective. In the same manner, while a high P_m can compromise the convergence of the procedure, a low P_m inhibits the search toward new zones. The best results are achieved with a medium population size (N=100-200), and by selecting the crossover and mutation probabilities in the ranges P_c=0.5-0.9 and P_m=0.01-0.05, respectively. Moreover, the highest achievable degree of satisfaction (degree of membership) for the given constraints and objectives is 0.9369, i.e. the best compromise solution due to the competing objectives.

Table 3. The design parameter values obtained from the MFGA optimization

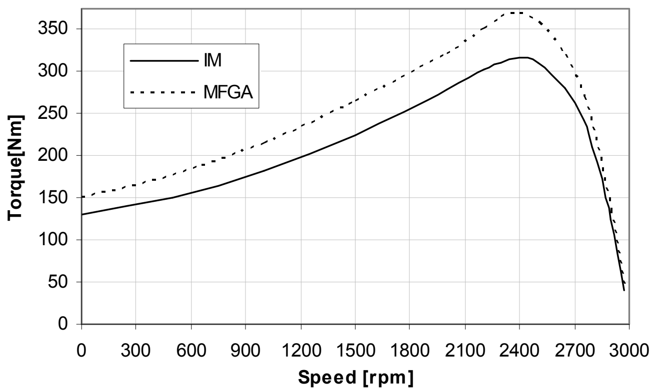
Design parameter	Industrial motor	Optimized motor
Number of turns per phase	36 (parallel)	36
Stator iron length (cm)	77	70.251
End-ring width (mm)	28	18.500
Stator interior diameter (cm)	8.925	9.645
Stator slot height (mm)	21.5	16.137
Stator slot width (mm)	13	13.138
Air-gap (cm)	0.075	0.0747
Bridge thickness of rotor closed slot (cm)	0.05	0.0639
Rotor bar diameter (mm)	8	8.823
Stator exterior diameter (cm)	18	18

Table 4. Comparisons of the different designs (Slip=0.05)

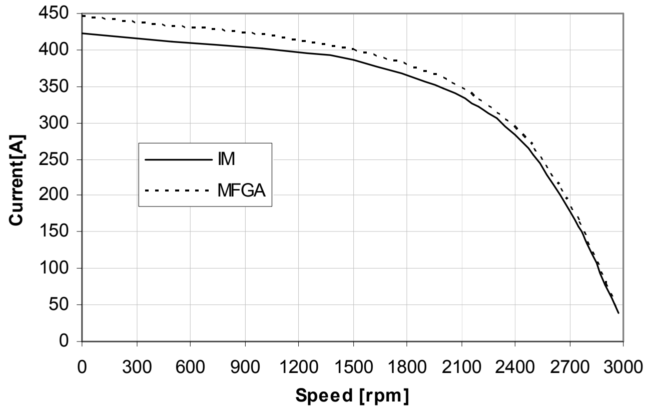
	Industrial motor	MFGA optimization
Full load torque (Nm)	**171.38**	**187.14**
Pull-out torque (Nm)	344.44	373.96
Starting torque (Nm)	132.22	153.29
Power factor	0.837	0.855
Efficiency (%)	85.59	86.96
Full load current (A)	106.37	109.37
Starting current (A)	422.28	450.80
Stator tooth flux density (T)	1.593	1.995
Rotor tooth flux density (T)	1.439	1.381
Stator yoke flux density (T)	1.519	1.544
Rotor yoke flux density (T)	2.13	1.540
Rotor current density (A/mm^2)	13.08	11.969
Temperature (^{0}C)	65	65
Stator slot filling factor	0.1898	0.182
Cost ($)	**807.72**	**756.00**

The results of the industrial motor and the MFGA optimization for submersible induction motor are given in Table 3 and 4. According to Table 4, while achieving performance improvements, the manufacturing cost of the motor is reduced by 7%. On the other hand, the starting torque and the full load torque are desirably increased by 15% and 9%, respectively, which are a remarkable increase. An essential remark here is that temperature rise of the motor is not known initially. Therefore, a fix value is given to the program. In view of the results, it is concluded that the MFGA optimization is suitable and can reach successful designs with lower cost and higher torque compared with the industrial motor while satisfying almost every constraint. Also, it was shown that MFGA optimization concludes with a good performance regarding the cost of different components and their dependencies on region, manufacturer and time. However, it is important to notice that while the performance improved, the efficiency and the power factor of the motor increased which shows additional capabilities of the optimization process.

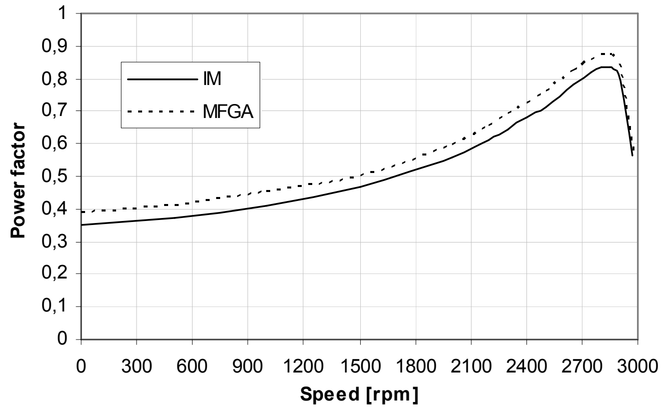
Fig. 3 depicts examples of performance characteristics of an industrial motor and the MFGA optimization as a function of the speed. The figures show a remarkable performance improvement on the optimized motor with respect to the industrial motor.



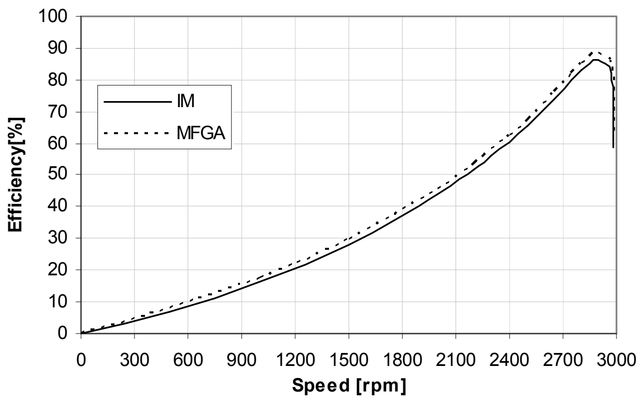
(a)



(b)



(c)



(d)

Figure 3. Performance Characteristics; a) Torque-speed, b) Current-speed, c) Power factor-speed and d) Efficiency-speed curves. (IM: Industrial motor, MFGA: Multiobjective fuzzy genetic algorithm optimization)

5. CONCLUSIONS

In this paper, an approach for multiobjective design optimization of the submersible induction motor utilizing the concept of fuzzy sets, convex fuzzy decision-making and a genetic algorithm has been presented. The multiobjective fuzzy optimization technique based on GA has been successfully applied for the optimal design of the submersible induction motor. The computer simulation results have shown the effectiveness of the proposed method. While the manufacturing cost decreased by 7%, the full load torque increased by 9% that shows remarkable results.

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LIST OF SYMBOLS

A_{1m}, A_b : cross-sectional area of stator and rotor conductor, respectively
A_r, A_g : cross-sectional area of end-ring and air-gap, respectively
Cu_{cost} : cost of unit weight of copper
D_e : stator diameter at centers of stator slots
D_o : stator outer diameter
D_r : rotor diameter
Fe_{cost} : cost of unit weight of iron
f_{ew} : end winding factor
L_1, L_2 : axial length of stator and rotor, respectively
m : number of phase
P_{fe} : density of the iron sheet
P_{sw}, P_{rw}: density of stator and rotor conductors, respectively
s :slip
SF : stacking factor
S_1, S_2 : number of stator and rotor slot, respectively
w_a, w_r : rotor end rings axial and radial width, respectively